

Prox Calculus Rules

$f(\mathbf{x})$	$\text{prox}_f(\mathbf{x})$	assumptions
$\sum_{i=1}^m f_i(\mathbf{x}_i)$	$\text{prox}_{f_1}(\mathbf{x}_1) \times \cdots \times \text{prox}_{f_m}(\mathbf{x}_m)$	
$g(\lambda \mathbf{x} + \mathbf{a})$	$\frac{1}{\lambda} [\text{prox}_{\lambda^2 g}(\mathbf{a} + \lambda \mathbf{x}) - \mathbf{a}]$	$\lambda \neq 0, \mathbf{a} \in \mathbb{E}, g$ proper
$\lambda g(\mathbf{x}/\lambda)$	$\lambda \text{prox}_{g/\lambda}(\mathbf{x}/\lambda)$	$\lambda \neq 0, g$ proper
$g(\mathbf{x}) + \frac{c}{2} \ \mathbf{x}\ ^2 + \langle \mathbf{a}, \mathbf{x} \rangle + \gamma$	$\text{prox}_{\frac{1}{c+1}g}(\frac{\mathbf{x}-\mathbf{a}}{c+1})$	$\mathbf{a} \in \mathbb{E}, c > 0, \gamma \in \mathbb{R},$ g proper
$g(\mathcal{A}(\mathbf{x}) + \mathbf{b})$	$\mathbf{x} + \frac{1}{\alpha} \mathcal{A}^T (\text{prox}_{\alpha g}(\mathcal{A}(\mathbf{x}) + \mathbf{b}) - \mathcal{A}(\mathbf{x}) - \mathbf{b})$	$\mathbf{b} \in \mathbb{R}^m, \mathcal{A} : \mathbb{V} \rightarrow \mathbb{R}^m,$ g proper closed convex, $\mathcal{A} \circ \mathcal{A}^T = \alpha I, \alpha > 0$
$g(\ \mathbf{x}\)$	$\text{prox}_g(\ \mathbf{x}\) \frac{\mathbf{x}}{\ \mathbf{x}\ }, \quad \mathbf{x} \neq \mathbf{0}$ $\{\mathbf{u} : \ \mathbf{u}\ = \text{prox}_g(0)\}, \quad \mathbf{x} = \mathbf{0}$	g proper closed convex, $\text{dom}(g) \subseteq [0, \infty)$