Feasible Net Income Distributions Under Income Tax Evasion:
An Equilibrium Analysis


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Abstract

We investigate how redistribution of income is affected by the fact that income is privately observed and agents may not be truthful in their reports to the tax authorities. In response, the government establishes an audit mechanism with penalties. Adhering to a signaling equilibrium concept, we prove that agents resort to mixed strategies, which makes it difficult for tax authorities to identify the true types. The audit strategy has a cut-off property: all income declarations below the pivotal income are audited with a constant probability. Other declarations are not audited. In spite of not being necessarily truthful, agents whose true income is below or equal the pivotal income, pay their liability and consequently, the government is implementing the designated tax schedule for these agents. Any effective tax schedule can be implemented by pushing the pivotal income upward, by setting a sufficiently high penalty on underreporting. In such a case, the effect of private information is only in reducing government revenues. In equilibrium, penalties and tax corrections equal the audit cost. Consequently, the audit system does not contribute directly to revenues, and its role is restricted to supporting the equilibrium.

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Introduction

Income tax evasion is a major problem in Western (and probably other) economies. It imposes constraints on the attainment of major goals of public policy such as the redistribution of income and efficient financing of government spending. In considering obstacles on the road to the economic union of the European Community references were made to differences in income tax compliance across member states (Germany and Holland versus France, Italy and other countries). In an attempt to shed some light on some unexplored aspects of this problem, we investigate in this paper the restrictions imposed by income tax evasion on the ability of governments to redistribute income.

The attempts of economists to model income tax evasion go back to the papers by Becker (1968) and Stigler (1970) who contended that criminal activities can be viewed as a rational behavior and should be analyzed as such. Given this conceptual framework, many authors (see, Allingham and Sandmo (1972), Srinivasan (1973), Stern (1978), Polinsky and Shavell (1979)) set up models and analyzed the optimality of various policy measures such as the penalty function and the probability of audit. A common feature of these papers is that they consider environments where agents have similar incomes which makes it impossible to consider many interesting questions. Models where agents have different incomes are considered in Rubinstein (1979), Landsberger and Meilijson (1982) and Greenberg (1984). However, as was correctly noted by Reinganum and Wilde (1985), these papers ignore the fact that in a taxation setting agents must send signals (income reports) about their privately known incomes. This is an important factor because even if agents misrepresent, their reports may have an informative value which should be accounted for. This contention lies at the heart of the more recent literature (see, Reinganum and Wilde (1985), (1986), Graetz, Reinganum and Wilde (1986), Border and Sobel (1987), Scotchmer (1987), Melumad and Mookherjee (1989), Cremer and Marchand (1990), Sanchez and Sobel (1993).

With the exception of GRW (1986) and RW (1986) who adhered to an equilibrium approach, the other papers adhered to the principal-agent framework, which makes the assumption that the principal has a commitment power. This is very unlikely in the case of tax auditing since, after income reports have been obtained, the principal may have good reasons to deviate from his previously announced policy and there seems to be no way for taxpayers to verify it, (see, GRW (1986), RW (1986) and MM (1989). Moreover, income tax authorities don’t usually announce what is their audit policy.

While trying to compare our results with those derived in other equilibrium models, we must exclude the paper by GRW (1996) since they considered a model where agents had two income levels only. Under such conditions it is impossible to address the problems investigated in our paper. Hence, a meaningful comparison is possible only with RW (1986); from a technical point of view, both models constitute signaling games. The major difference between RW (1986) and our paper is that they considered a separating equilibrium whereas we have excluded this type of equilibrium since it implies that after reports have been submitted, the tax authorities know
agents’ true incomes, which in our opinion is at variance with reality.

It is interesting to notice that the signaling game which is induced by the problem we are addressing, is structurally different from most signaling games that consider economic problems. In these games, one can naturally refer to ‘strong’ versus ‘weak’ types, where the latter usually gain if there is no separation and the strong types are worse off. Therefore, it is in the best interest of the strong types to signal in order to separate themselves; see, Spence (1974), Rothschild and Stiglitz (1976), Milgrom and Roberts (1982). This is why in these situations separating equilibria make sense. In our case, however, there is no natural distinction between strong and weak types and it is not clear which types, if any, want to separate themselves. Unlike in the other models, in the case of taxation signaling is imposed and does not reflect the interest of any type. These important differences illustrate once more why, in spite of the fact that separating equilibria are common in economic models, this should not be an argument in support of this equilibrium concept in the environment we consider.

The major problem we address, is how do the constraints imposed by the fact that income is agents’ private information, affect government’s ability to implement a given income tax schedules. We prove that in equilibrium agents resort, generically, to mixed strategies. The audit strategy has a cut-off property: the auditor audits with a constant probability all declarations below a given reported income – the pivotal income. Other income reports are not audited at all. This is in contrast with results obtained for separating equilibria (see [14]) where the audit policy does not have the cut off property and the audit probability is decreasing with reported income.

In spite of not being truthful, in equilibrium, types bellow the pivotal income pay their liability; namely, for these types the government is implementing the income distribution defined by the tax schedule. Consequently, if the pivotal income is sufficiently high, the government is implementing the tax schedule it wanted; The only effect of the asymmetric information is that government revenues are smaller. The lower is the pivotal income, the larger is the deviation from the desired effective tax schedule since types above the pivotal income pay less than their liability. These results are almost opposite to those established by [14] who proved that tax evasion is decreasing with true income; the different results are due to different concept of equilibrium adopted in the two papers.

We also prove that, in equilibrium, penalties collected from agents who were audited plus the tax corrections, equal the auditing cost. Namely, the government does not benefit directly from penalizing agents who were found underreporting their incomes.

The paper is organized as follows. In sections 2 and 3 we introduce the model and its basic properties. In section 4 we introduce the concept of the bureaucratically efficient equilibrium. In Section 5 we prove several properties of equilibrium. In particular we prove that there exists a bureaucratically efficient equilibrium, and that every such equilibrium is sequential (see [8]). In section 6 we present and discuss our main results. We also provide in this section a numerical example, that illustrates some of our conclusions. In section 7 we prove that agents use, generically, mixed strategies. In Section 8 we evaluate our major results.
2 The Model

We assume that tax payers’ income is described by a random variable $X$ which takes values from a finite set $\{x_1, x_2, \ldots, x_N\}$. For convenience we let $0 < x_1 < x_2 < \cdots < x_N$. We refer to a tax payer whose true income is $x_i$ as type- $i$. Probabilities associated with types are denoted by $h_i > 0$; that is, $P(X = x_i) = h_i$. Our model is analyzed as a two stage signaling game with two players: the principal and the agent who, eventually, becomes a type $k$ for some $k = 1, 2, \ldots, N$. After having 'received' his income, which is private information, the tax payer reports to the principal of his type. This report may be based on lotteries, one for each type, which are determined by the agent as a best response to the principal’s audit system as assessed by the agent.

Agents are required to pay an income tax according to a tax function $t(x_i) = t_i$, where

$$t_1 < t_2 < \cdots < t_N. \tag{2.1}$$

When type-$k$ announces his type, say, $i$, the principal can decide to audit or not. If an audit does not take place the principal collects $t_i$. If an audit takes place the principal entails a cost $c_k$ which depends on the true type. Hence, at the stage when the auditing decision takes place, the principal does not know the audit cost. In order to capture the notion that it is more expensive to audit large incomes, we assume that $0 < c_1 \leq c_2 \leq \cdots \leq c_N$. When an audit takes place true income is revealed and if the agent misrepresented he is required to pay a penalty and the tax on his true income. Let $f_{k,i}$ stand for the penalty to be paid by type-$k$ who reported being type-$i$. We assume the following structure of the penalty function:

$$f_{k,i} = \alpha(t_k - t_i), \quad \text{if } i < k \quad \text{and } f_{k,i} = 0 \text{ if } i \geq k, \tag{2.2}$$

where $\alpha > 0$ is a fixed but arbitrary constant. Furthermore,

$$t_k + f_{k,i} - c_k > t_i \quad \text{for all } k > i. \tag{2.3}$$

(2.3) implies that it is always worthwhile to audit an income report known not to be true. This condition eliminates the possibility of obtaining separating equilibria. The latter are not particularly appealing in a tax evasion model because they imply that after income declarations have been made the auditor knows the true income of each tax payer which, we believe, is counterfactual.

Some readers may be unhappy with an exogenously determined penalty function; so are we. However, to make the penalty a strategic variable would require a definition of the feasible set from which such functions can be chosen. With unbounded penalties, even a first best solution may be achieved and therefore the definition of the feasible set is very important. In order to comply somehow with reality while thinking of how to define this feasible set, one must respect considerations such as 'fairness' and requirements that punishments be 'proportional' to offences which play an important role in the judicial system and therefore can not be ignored. Consequently, we feel that trying to accommodate these considerations would constitute a significant deviation from the main course of this paper and therefore we chose an exogenous penalty function which, at least in tax evasion situations, may not be very far from common practice. Another reason for not making penalties and taxes strategic variables is that, as contended by RW (1986), these instruments are determined at higher levels of government and are therefore
exogenous as viewed by the tax collecting agency. An interesting analysis of the reasons why governments delegate the authority to collect taxes to subordinates is provided by MM (1989).

**The Principal’s Strategies**

The principal adheres to a behavioral strategy represented by a vector

\[ p = (p_1, p_2, \ldots, p_N), \]  

(2.4)

where \( 0 \leq p_i \leq 1 \) stands for the probability of an audit if an announcement \( x_i \) was made.

**Agents’ Strategies**

When an agent is assigned his type, he must inform the principal. In doing so, we allow him resort to randomized reporting schemes. Namely, each type-\( k \)-designs a probability vector, \( \beta_k = (\beta_{k,i})_{i=1}^{N} \), where \( \beta_{k,i} \geq 0 \) stands for the probability that type-\( k \) declares himself as being type-\( i \). Hence, \( \sum_{i=1}^{N} \beta_{k,i} = 1 \). Without loss of generality we assume that an agent does not place a positive probability on types which are greater than his type. Hence, we assume:

\[ \sum_{i=1}^{k} \beta_{k,i} = 1 \quad \text{for every } 1 \leq k \leq N. \]  

(2.5)

Every vector \( \beta = (\beta_1, \beta_2, \ldots, \beta_N) \) is called a strategy for the agent. Every \( \beta_k \) is called a strategy for type-\( k \).

### 3 Payoffs and Bayesian Beliefs

Let \( h_i(\beta) \) stand for the probability that \( x_i \) is announced when the agent uses the strategy \( \beta \). That is,

\[ h_i(\beta) = \sum_{k=1}^{N} \beta_{k,i} h_k. \]  

(3.1)

**Definition 1** We say that \( i \) (or \( x_i \)) is in the support of the agent’s strategy \( \beta = (\beta_1, \beta_2, \ldots, \beta_N) \) if at least one type assigns to \( x_i \) a positive probability, that is if there exists \( i \leq k \leq N \) such that \( \beta_{k,i} > 0 \). To this effect we use the notation \( i \in S(\beta) \).

Hence, \( i \in S(\beta) \) if and only if \( h_i(\beta) > 0 \). If the principal believe that the agent is using \( \beta \), and the reported income is \( x_i \), \( i \in S(\beta) \), he can apply Bayes formula to update his prior beliefs about the true type. Let \( q_\beta(k|i) \) stands for the probability that the true type is \( k \), given that the announcement \( i \) was made by an agent that uses \( \beta \). That is,

\[ q_\beta(k|i) = \frac{\beta_{k,i} h_k}{h_i(\beta)}, \quad k = i, i+1, \ldots, N. \]  

(3.2)

(3.2) is not defined for \( i \notin S(\beta) \).

**Expected Payments Made by Agents**

We use the following notations: Let \( A_{k,i}(p_i) \) denote the expected payment of type-\( k \) who announced that his type is \( i \), and given that having heard \( x_i \), the principal audits with probability \( p_i \). Hence,

\[ A_{k,i}(p_i) = (1 - p_i)t_i + p_i(t_k + f_{k,i}). \]  

(3.3)
Let $A_k(\beta_k, p)$ denote the expected payment of a type-$k$ if he uses $\beta_k$ and the audit probabilities are $p = (p_1, p_2, \cdots, p_N)$. That is

$$A_k(\beta_k, p) = \sum_{i=1}^{k} A_{k,i}(p_i)\beta_{k,i}. \tag{3.4}$$

Hence, by (3.3)

$$A_k(\beta_k, p) = \sum_{i=1}^{k} ((1 - p_i)t_i + p_i(t_k + f_{k,i}))\beta_{k,i}. \tag{3.5}$$

Finally, let $A(P, \beta)$ denote the ex-ante expected payment made by an agent (i.e. before he knows his type and, obviously, prior to his report). That is,

$$A(\beta, p) = \sum_{k=1}^{N} A_k(\beta_k, p)h_k. \tag{3.6}$$

**Expected Receipts of The Principal**

Let $E_{k,i}(p_i)$ denote the expected net receipts accruing from type-$k$ who declares that he is a type-$i$, that is

$$E_{k,i}(p_i) = (1 - p_i)t_i + p_i(t_k + f_{k,i}) - p_i c_k. \tag{3.7}$$

Let $E_i(\beta, p_i)$ denote the expected net receipts of the principal accruing from an agent who announced $x_i$, provided that $i \in S(\beta)$, that is

$$E_i(\beta, p_i) = \sum_{k=i}^{N} ((1 - p_i)t_i + p_i(t_k + f_{k,i}) - p_i c_k)q_{\beta}(k|i). \tag{3.8}$$

Let $E(\beta, p)$ denotes the expected net receipts of the principal;

$$E(\beta, p) = \sum_{i \in S(\beta)} E_i(\beta, p_i)h_i(\beta). \tag{3.9}$$

Upon inserting (2.2) into (3.3) and (3.8) we obtain

$$A_{k,i}(p_i) = t_k + (1 + \alpha)(t_k - t_i)(p_i - \frac{1}{1 + \alpha}), \tag{3.10}$$

and for $i \in S(\beta)$

$$E_i(\beta, p_i) = t_i + p_iK_i(\beta), \tag{3.11}$$

where,

$$K_i(\beta) = -c_iq_{\beta}(i|i) + \sum_{k=i+1}^{N} [(t_k - t_i)(1 + \alpha) - c_k]q_{\beta}(k|i). \tag{3.12}$$

Note that upon receiving a signal $i$, the principal’s expected net gain can be decomposed into an immediate gain, $t_i$, which is always realized and will be the final gain if no audit takes place, and a second component $K_i(\beta)$ which takes place if there is an audit.
4 Equilibrium

A pair of strategies \((\beta^*, p^*)\) is a Nash equilibrium if

\[
E(\beta^*, p^*) \geq E(\beta^*, p) \quad \text{for all } p, \quad (4.1)
\]

and

\[
A(\beta^*, p^*) \leq A(\beta, p^*) \quad \text{for all } \beta. \quad (4.2)
\]

Condition (4.1) says that when using \(p^*\) the principal maximizes ex-ante against \(\beta^*\). This condition is equivalent to the condition that for every \(i\) which is consistent with the agent’s strategy \(\beta\), the principal, after having received the signal \(i\) maximizes against \(\beta^*\) by using \(p_i\). That is (4.1) can be replaced by

\[
E_i(\beta^*, p_i^*) \geq E_i(\beta^*, p_i) \quad \text{for all } i \in S(\beta^*) \text{ and for all } p_i \in [0, 1]. \quad (4.3)
\]

Similarly, condition (4.2) can be replaced by

\[
A_k(\beta^*, p^*) \leq A_k(\beta_k, p^*) \quad \text{for all } k \text{ and for all } \beta_k. \quad (4.4)
\]

A refinement of the Nash equilibrium to which we adhere in this paper is introduced in the following Definition:

**Definition 2** A Nash equilibrium \((\beta^*, p^*)\) is bureaucratically efficient if it is impossible to lower one or more audit probabilities, not changing the other, and remain in equilibrium with the initial \(\beta^*\).

Adherence to this notion of equilibrium relies on the contention that if there are two Nash equilibria \((\beta^*, p^*)\) and \((\beta^{**}, p^{**})\) such that \(p^* \leq p^{**}\) with a strict inequality for at least one \(i\), then bureaucrats who are in charge of audits will choose \(p^*\). Namely, all other things being equal administrators wish to audit less. This is an arbitrary assumption as are many refinements suggested for signaling games. We believe that this assumption is reasonable although one could think of other considerations as well. For example, one might believe in an equilibrium which corresponds to the lowest expected cost of audit. But then, since the choice is between equilibria which generate the same expected net revenue, this ‘efficiency’ argument could also be questioned. In Proposition 3 in Section 5 we show that a bureaucratically efficient equilibrium always exists, and every such equilibrium has the desired property that it is a sequential equilibrium.

5 Properties of Bureaucratically Efficient Equilibria

In this section we provide several properties of equilibrium and of bureaucratically efficient equilibrium in our Model. Some of these properties are only needed for the proof of our main theorem, Theorem 1, and some are of independent interest. The proofs of the Propositions stated in this section are given in the Appendix.

**Lemma 1** Let \(p = (p_1, p_2, \ldots, p_N)\) be a vector of audit probabilities and assume that \(p_i > 1/(1 + \alpha)\) for some \(i\). Let \(\beta_k, k > i\), be a best response of type-\(k\) against \(p\). Then, \(\beta_{k,i} = 0\).
Proof  By (3.10) $A_{k,k}(p_k) < A_{k,i}(p_i)$ for all $k > i$, which shows that declaring $x_i$ is not a best response of type-$k$. It is well known that in a mixed best response strategy a player assigns a positive probability only to a pure best response strategy. Hence, the result follows.

Proposition 1 Let $(\beta^*, p^*)$ be a Nash equilibrium, and let $i \in S(\beta^*)$. Then,

(i) $p^*_i \leq \frac{1}{1+\alpha}$.
(ii) $K_i(\beta^*) \leq 0$.
(iii) If $p^*_i > 0$, $K_i(\beta^*) = 0$.
(iv) The expected net revenue of the principal accruing from an agent who declared $x_i$ is $t_i$, that is, $E_i(\beta^*, p^*_i) = t_i$

An intuitive explanation of (i) in Proposition 1 is as follows: If $p^*_i > \frac{1}{1+\alpha}$, then every type-$k$, whose true income exceeds $x_i$ never declare $x_i$ since as a sanctuary for those who consider to misrepresent, $i$ is too 'hot'. This is just a verbal statement of the fact that $A_{k,i}(p_i) > t_k = A_{k,k}(p_k)$ for all $k > i$, which follows from (3.10). Hence, any declaration $x_i$, if there is any, must be truthful. Therefore, the best response of the principal would have been to set $p_i = 0$ since it does not make sense to audit announcements which are truthful with a positive probability. This, obviously, contradicts $p^*_i > \frac{1}{1+\alpha}$. Claim (iii) illustrates the preventive role of the audit system which supports the equilibrium. After having collected the tax on the declared income, the additional expected revenues from collecting the true tax and the accruing penalties just cover the audit cost. Hence, in equilibrium, penalties contribute to revenues only indirectly by affecting the declaration strategy of agents.

In the sequel we shall need the following definition:

Definition 3 An income $x_s$, $1 \leq s \leq N$ is the pivotal income of the vector, $p$ of audit probabilities, or $s$ is the cut-off type, if $s$ is the lowest index, for which $p_s < \frac{1}{1+\alpha}$.

Proposition 2 Let $(\beta^*, p^*)$ be a bureaucratically efficient equilibrium. Then,

(i) $p^*_i \leq \frac{1}{1+\alpha}$ for every $1 \leq i \leq N$.
(ii) $p^*_N = 0$. Consequently, there exists a pivotal income for $p^*$.
(iii) Let $x_s$ be the pivotal income, then $\beta^*_{k,s} = 1$ for all $k > s$. That is, agents whose true income is above the pivotal income $x_s$ always report $x_s$.
(iv) All income reports higher or equal to the pivotal income $x_s$, are never audited.

We now show that the concept of a bureaucratically efficient equilibrium is not vacuous, and that such equilibria satisfy an attractive property.

Proposition 3 There exists a bureaucratically efficient equilibrium, and every such an equilibrium is sequential.

6 Bureaucratically Efficient Equilibria: the effective Income Tax schedule

The following Theorem summarizes the major results derived in this paper.

\footnote{Note that by (i) in Proposition 1 the claim holds for $i \in S(\beta^*)$.}
Theorem 1 Let \((\beta^*, p^*)\) be a bureaucratically efficient equilibrium. Then,

(a) There exists a pivotal income \(x_s\) with \(1 \leq s \leq N\). Moreover, \(p^* = (1/1+\alpha), (1/1+\alpha), \ldots, (1/1+\alpha), 0, 0, \ldots, 0)\), where the first 0 in \(p^*\) appears in the \(s^{th}\) place.

(b) Every type-\(k\) whose income exceeds the pivotal income \(x_s\) declares \(x_s\) with probability 1. That is, \(\beta^*_{k,s} = 1\) for all \(k > s\).

(c) The expected net revenue of the principal from an agent who declares \(x_i\) is \(t_i\). Namely, \(E_i(\beta^*, p^*) = t_i\) for every \(i \in S(\beta)\).

(d) The expected tax payment of the types whose incomes are smaller or equal to the pivotal income \(x_s\) equal their liability. That is, \(A_k(\beta^*_{k}, p^*) = t_k\) for every \(k \leq s\).

(e) The expected tax payments of the types whose incomes are above the pivotal income are equal to the liability of type-\(s\). Namely, \(A_k(\beta^*_{k}, p^*) = t_s\) for every \(k > s\).

Proof A pivotal income \(x_s\) exists by (ii) in Proposition 2. By Definition 3, \(p_i \geq 1/(1+\alpha)\) for every \(i < s\), and by (i) in Proposition 2, \(p_i \leq 1/(1+\alpha)\) for each such \(i\). Therefore, \(p_i = 1/(1+\alpha)\) for every \(i < s\). By (iv) in Proposition 2, \(p_i = 0\) for every \(i \geq s\). Therefore, (a) is established. (b) follows directly from (iii) in Proposition 2, (c) follows from (iv) in Proposition 1. As for (d), Let \(k \leq s\). Since for all \(i \leq k\), \(p_i* = 1/(1+\alpha)\), \(A_k,i(p_i) = t_k\) for all such \(i\) by (3.10). Therefore, the result follows from (3.4). As for (e), let \(k > s\). By (b), \(k\) declares \(x_s\) with probability 1, and by (a), with probability 1 he is not audited. Hence, he pays \(t_s\).

Statement (a) in the Theorem stipulates that the structural form of the principal’s auditing strategy is unique. Regardless of agents’ reports the principal never audits announcements which are higher or equal to the pivotal income, which is the highest declaration ever made in equilibrium. Income reports which are below the pivotal income are audited with a constant probability of \(1/(1+\alpha)\). The fact that the principal audits with a constant probability or does not audit can be interpreted as if he ignores most of the information which might be extracted from the signals. In conjunction with Proposition 4, in which we will show that, generically, agents do not use pure strategies, one can elaborate further on this result saying that by using mixed strategies the agents succeed in making their reports to be of almost no informative value to the principal. Note that auditing with the constant probability \(1/(1+\alpha)\) does not imply that the principal’s strategy in equilibrium is uniquely determined, since strategies can differ by having different pivotal incomes, as illustrated in Example 1.

Statement (b) in the Theorem claims that agents whose declarations are above the pivotal income misrepresent with probability one and the pattern of their misrepresentation is uniquely established: they all announce the pivotal income. Hence, when the pivotal income is announced it is very likely that it constitutes a misrepresentation, and nevertheless the principal never audits it. One may wonder how does this behavior of the principal constitute a best response to the agents’ strategies. The answer to this question is based on the result established in (iii) in Proposition 1, which claims that in equilibrium, if an announcement is audited with a positive probability then it must be the case that the expected net gain from this audit is zero. This result drives also (c) in the Theorem: If \(x_i\) is announced and the principal does not audit then he collects \(t_i\). If he audits then, by (iii) in Proposition 1, he also collects \(t_i\) hence, (c) follows.

Statement (d) in the Theorem claims that the expected payments of agents whose income are below the pivotal income are the same as their liability. Namely, they pay what they would have paid if they were truthful. This property allows the government to implement on these types any income distribution it desired. Private information affects only government revenue.
Of course, this may force the government to decide on a higher tax schedule. Statement (e) illustrates that the types above the pivotal income pay less than their liability. The significance of this conclusion depends on the location of the pivotal income. The higher it is, the smaller the set of types who pay less than they had to. A natural question is how does the pivotal income change if we modify parameters of the model. We checked the penalty effect, $\alpha$, and, not surprisingly, we found that raising $\alpha$ increases the pivotal income. Hence, if $\alpha$ is sufficiently high it may be possible to implement any increasing effective tax schedules. The problem is that in reality penalties are beyond the control of the tax collecting agency and, what is more important, social and political factors play a major role in their determination and therefore, even at the level of those who have the authority to change penalties, it may be difficult to have them do it.

**Example 1.**

There are four types $k = 1, 2, 3, 4$ with probabilities

$$h_1 = .55, \quad h_2 = .15, \quad h_3 = .5, \quad h_4 = .075.$$ 

The corresponding tax function is

$$t_1 = 2, \quad t_2 = 6, \quad t_3 = 12, \quad t_4 = 20.125,$$

the audit costs are:

$$c_1 = 1.2, \quad c_2 = 2, \quad c_3 = 3, \quad c_4 = 4,$$

and $a = .6$.

We computed two bureaucratically efficient equilibria for this example, $(\beta, p)$ and $(\beta^*, p^*)$, where,

$$\beta_1 = (1), \quad \beta_2 = (1, 0), \quad \beta_3 = (0, 0, 1), \quad \beta_4 = (0, 0, 1, 0),$$

$$p = (.625, .625, 0, 0),$$

$$\beta_1^* = (1), \quad \beta_2^* = (.3733), \quad \beta_3 = (.14, .13, .73), \quad \beta_4 = (0, 0, .73, .27),$$

and

$$p^* = (.625, .625, .625, 0).$$

In the equilibrium $(\beta, p)$, agents use pure strategies and the pivotal income is 3. In the equilibrium $(\beta^*, p^*)$, agents resort to mixed strategies and the pivotal income is 4.

**7 Agents’ Equilibrium Strategies**

In this section we prove that if the index of the pivotal income is greater than 1, then in a bureaucratically efficient equilibrium, if the pivotal income is not the lowest income, agents generically resort to mixed strategies. The reader may be unhappy with this result. However, we contend that in our model mixed strategies equilibria are reasonable. It is not necessarily interpreted as if individual taxpayers randomize (which is what may be disturbing about mixed strategies). Since in our model it is reasonable to assume that many taxpayers have similar incomes, it is possible that every taxpayer in an income group uses a pure strategy. This
behavior induces an 'aggregate' taxpayer who randomizes where aggregation is over taxpayers with similar incomes (recall that a type is defined by an income level)\(^2\).

Note that if \(x_1\) is the pivotal income, then we have the unlikely equilibrium, in which all agents report \(x_1\), and the principal never audit. To eliminate such an equilibrium we assume the following inequality:

\[
\sum_{k=1}^{N} h_k [t_k + f_{k,1} - c_k] > t_1, \tag{7.1}
\]

and we prove:

**Lemma 2** If (7.1) holds, then in every bureaucratically efficient equilibrium the index \(s\) of the pivotal income is greater than 1.

**Proof:**

If \(s = 1\), then by (4) in Proposition 2, \(p_j^* = 0\) for all \(1 \leq j \leq N\), and by (iii) in Proposition 2, all types declare \(x_1\) with probability 1. However, if all types declare \(x_1\), then by (7.1) the government would be better off if it audits the declaration \(x_1\). A contradiction. Therefore \(s > 1\).

**Proposition 4** Let all the parameters of the model be fixed except for the prior probabilities of incomes, and let \(D\) be the set of all possible distributions \(h = (h_1, h_2, \ldots, h_N)\) with \(h_k > 0\) for all \(1 \leq i \leq n\), that satisfy (7.1). Let \(\lambda\) be the Borel probability on \(D\). Suppose we pick at random from \(D\) a distribution \(h\). Then, with probability 1, in every bureaucratically efficient equilibrium agents resort to mixed strategies. That is, at least one type uses a mixed strategy.

**Proof** Note that \(D\) is a convex set of dimension \(N - 1\). Let \(D_p\) be the set of all \(h \in D\) for which there exists a bureaucratically efficient equilibrium \((\beta^*, p^*)\) with \(\beta^*\) a pure strategy. We will show that \(D_p\) is contained in the union of a finite number of convex subsets of \(D\), each of dimension of at most \(N - 2\). Therefore, \(\lambda(D_p) = 0\).

Indeed, For every \(1 \leq i < N\) and for every \(S\) which is contained in \(\{i + 1, i + 2, \ldots, N\}\), define

\[
D(i, S) = \{h \in D : -h_i c_i + \sum_{k \in S} [(t_k - t_i)(1 + \alpha) - c_k] h_k = 0\}.
\]

Obviously, \(D(i, S)\) is a convex subset of \(D\) whose dimension is at most \(N - 2\). We complete the proof by showing that

\[
D_p \subseteq \bigcup_{i=1}^{N} \bigcup_{S \subseteq \{i+1, \ldots, N\}} D(i, S).
\]

Let \(h \in D_p\), and let \((\beta^*, p^*)\) be a bureaucratically efficient equilibrium with respect to \(h\), in which \(\beta^*\) is a pure strategy. Denote by \(x_s\) the pivotal income (see Theorem 1). By Lemma 2, \(s > 1\). Let \(i \in S(\beta^*)\) with \(i < s\). Since \(p_i = \frac{1}{(1 + \alpha)\alpha}\), (3) of Proposition 1 implies that \(K_i(\beta^*) = 0\). Let \(S\) be the set of all type \(-k, k > i\), who declare \(x_i\). As \(K_i(\beta^*) = 0\)

\[
-h_i c_i + \sum_{k \in S} [(t_k - t_i)(1 + \alpha) - c_k] h_k = 0.
\]

\(^2\)Since our model is discrete, this interpretation of mixed strategies is valid only under appropriate technical assumptions. More precisely, the numbers \(\beta_{k,i}\) should be rational, that is \(\beta_{k,i} = \frac{r_{k,i}}{q_{k,i}}\), for all \(i \leq k \leq N\), where \(r_{k,i}\) and \(q_{k,i}\) are natural numbers, and \(q_{k,i}\) should divide the total number of agents with income \(x_i\).
Hence, \( h \in D(i, S) \). □

Note that the result stated in Proposition 4 is generic. Hence, it does not imply that there can not be a bureaucratically efficient equilibrium in pure strategies. An example of a bureaucratically efficient equilibrium where agents uses pure strategy is presented in Example 1.

8 Evaluation Of The Main Results

The main result of our paper is that when income is privately observed, agents whose income is below or equal to a pivotal income, pay their liability. Hence, they do not gain from not being truthful. Consequently, for these types, the government is implementing the effective tax schedule. If the pivotal income is sufficiently high, not being truthful does not interfere with the attainment of any desired income distribution. The only effect of private information is that it makes tax collection costly.

In equilibrium, agents resort to mixed strategies when reporting their incomes, which makes their reports of little informative value; high income agents can always mimic low income agents’ signals. True, the cost of mimicking is increasing with the amount evaded, but the latter does not necessarily increase with true income.

The auditing policy has two distinct features: (i) Auditing takes place with a constant probability, and (ii) declarations which are above a given level, the pivot income, or equal to it, are not audited. The result stated in (i) can be attributed to the fact that agents are using mixed strategies, which leaves the announcements with very little informative value and there is no reason to condition auditing on income declarations.

We should remember when trying to confront our results with reality, that political considerations have a strong impact on public policy. Hence, the results we derived constitute, only one components of the system that drives reality.

9 Appendix

Proof of Proposition 1

(i) Suppose, in negation, that \( p^*_{i} > \frac{1}{1+\alpha} \). By Lemma 1, every type-\( k \), \( k > i \), assigns probability zero to \( x_i \), and therefore, \( q_{\beta^*}(i|i) = 1 \), since \( i \in S(\beta^*) \). Hence, by (3.11) and (3.12), \( E_i(\beta^*, p_i) = t_i - p_i c_i \). This implies that the principal’s best response against \( \beta^* \) is \( p_i^* = 0 \), contradicting \( p_i^* > \frac{1}{1+\alpha} \).

(ii) Since \( (\beta^*, p^*) \) is a Nash equilibrium and \( i \in S(\beta^*) \), then, by (4.3) \( E_i(\beta^*, p_i^*) \geq E_i(\beta^*, p_i) \) for every \( p_i \in [0, 1] \). In particular, the last inequality holds for \( p_i = 1 \). Therefore, by (3.11) and (3.12), \( t_i + p_i^* K_i(\beta^*) \geq t_i + K_i(\beta^*) \). By (1) of this Proposition, \( p_i^* < 1 \) and therefore, the last inequality implies \( K_i(\beta^*) \leq 0 \).

(iii) The proof is very similar to the one we used in the proof of (ii), and therefore we omit it.

(iv) Follows directly from (3.11) and (iii). □

Proof of Proposition 2

(i) By (i) in Proposition 1, if \( i \in S(\beta^*) \), the result holds for every Nash equilibrium and in particular it is true for a bureaucratically efficient equilibrium. Hence, we have to prove the
Lemma for \( i \notin S(\beta^*) \). Let then \( i \notin S(\beta^*) \), that is \( h_i(\beta^*) = 0 \), and assume in negation that \( p_i^* > \frac{1}{(1 + \alpha)} \). Let \( \varepsilon > 0 \) be small enough so that \( p_i^* - \varepsilon > \frac{1}{(1 + \alpha)} \), and define the vector of probabilities

\[
p = (p_{i1}^*, p_{i2}^*, \ldots, p_{i-1}^*, p_i^* - \varepsilon, p_{i+1}^*, \ldots, p_N^*).
\]

We will show that \((\beta^*, p)\) is a Nash equilibrium contradicting the contention that \((\beta^*, p^*)\) is bureaucratically efficient. Obviously, \( p \) is a best response against \( \beta^* \), since changing a response to a signal which is not in the support of \( \beta^* \) does not change the expected payoffs of the principal.

What remains to be shown is that for every \( A \neq i \), \( \beta_k^* \) is a best response against \( \beta^* \). Indeed, if \( k \leq i \), then changing \( p_i^* \) has no effect on the expected payment of type-\( k \) and therefore \( \beta_k^* \) is a best response to \( p \). If \( k > i \), lowering \( p_i^* \) may tempt type-\( k \) to declare \( i \) with a positive probability and thus changing his best response. To show that this does not happen, denote by \( \beta_k \) a best response of type-\( k \) against \( P \) and assume it is strictly better than \( \beta_k^* \). That is,

\[
A_k(\beta_k, p) < A_k(\beta_k^*, p^*).
\]

Stating (9.2) explicitly by using (3.5) and (2.2) we obtain:

\[
\sum_{j=1}^{k} [t_k + (1 + \alpha)(t_k - t_j)](p_j - \frac{1}{(1 + \alpha)})((\beta_k)_{j}) < \sum_{j=1}^{k} [t_k + (1 + \alpha)(t_k - t_j)](p_j - \frac{1}{(1 + \alpha)})((\beta_k^*)_{j}).
\]

Since \( p_i > \frac{1}{(1 + \alpha)} \), \( \beta_k = 0 \) by Lemma 1, and since \( i \notin S(\beta^*) \), \( \beta_k^* = 0 \). Therefore, the inequality (9.3) is independent of the value taken by the audit probability of \( i \). Since \( p_j = p_j^* \) for every \( j \neq i \), the inequality above is valid when we replace \( p_j \) by \( p_j^* \) for all \( j \), which contradicts \( \beta_k^* \) being a best response against \( p^* \).

(ii) Suppose, in negation, that \( p_N^* > 0 \). As in the proof of (1) it can be shown that \((\beta^*, p)\) is also an equilibrium, where \( p \) is obtained from \( p^* \) by replacing \( p_N^* \) with \( p_N^* - \varepsilon \). This is in contradiction with \((\beta^*, p^*)\) being a bureaucratically efficient equilibrium.

(iii) We first prove the assertion for type-(\( s + 1 \)). Since for all \( j < s \), \( p_j^* = \frac{1}{(1 + \alpha)} \), (3.9) implies \( A_{s+1,j}(p_j^*) = t_{s+1} \) for \( j < s \). Obviously, for \( j = s + 1 \), \( A_{s+1,s+1}(p_{s+1}^*) = t_{s+1} \). Since \( p_s^* < \frac{1}{(1 + \alpha)} \), \( A_{s+1,s}(p_s^*) < t_{s+1} \). Hence, declaring \( x_s \) is the unique best response of type-(\( s + 1 \)) against \( p^* \).

Consider now type-(\( s + 2 \)). As before, we can prove that declaring \( x_s \) is strictly better for type-(\( s + 2 \)) than declaring \( x_j \), for all \( j < s \), and it is also better than being truthful. Hence, \( \beta_{s+2}^* = 0 \) for \( j < s \) and for \( j = s + 2 \). It remains to show that \( \beta_{s+2,s+1}^* = 0 \). Suppose, in negation, that \( \beta_{s+2,s+1}^* > 0 \). Then, \( (s + 1) \) is in the support of \( \beta^* \) and therefore, by (ii) of Proposition 1, \( K_{s+1}(\beta^*) \leq 0 \). We have already shown that \( \beta_{s+1,s+1}^* = 0 \). Therefore, \( q_{\beta^*}(s+1|s+1) = 0 \), which implies, by (3.11), that \( K_{s+1}(\beta^*) = \sum_{k=s+2}^{N} q_{\beta^*}(k|s+1)\delta_k \), where \( \delta_k = (1 + \alpha)(t_k - t_{s+1}) - c_k \).

By (2.3), \( \delta_k > 0 \) for all \( k \), and therefore \( K_{s+1}(\beta^*) \geq q_{\beta^*}(s+2|s+1)\delta_k + 2 > 0 \), which contradicts \( K_{s+1}(\beta^*) \leq 0 \). Therefore, \( \beta_{s+2,s+1}^* = 1 \). We can similarly prove, by induction, that \( \beta_{k,s+1}^* = 1 \) for all \( k > s \).

(iv) We have already proved that \( p_N^* = 0 \) and therefore we can assume that \( s < N \). We first prove that \( p_s^* = 0 \). Suppose in negation that \( p_s^* > 0 \). Since every type-\( k \), \( k > s \), declares \( x_s \), \( s \) is in the support of \( \beta^* \) and therefore, by (iii) in Proposition 1, \( K_s(\beta^*) = 0 \). Let \( p \) be obtained from \( p^* \) by decreasing \( p_s^* \) to \( p_s^* - \varepsilon > 0 \), and keeping all other audit probabilities
unchanged. We will show that \((\beta^*, p)\) is a Nash equilibrium which is in contradiction with the contention that \((\beta^*, p^*)\) is bureaucratically efficient. Applying \(K_1(\beta^*) = 0\) and (3.11) implies that \(E_s(\beta^*, p_s^* - \varepsilon) = E_s(\beta^*, p_s^*) = t_s\). Hence, \(p^*\) is a best response against \(\beta^*\) as much as \(p^*\) is. On the other hand, \(\beta^*\) is a best response against \(p\) because every type-\(k\), \(k > s\) declares \(x_s\) with probability 1 (and decreasing the audit probability of type-\(s\) increases the motivation to declare \(x_s\)). We proceed to show that \(p_j^* = 0\) for all \(s < j < N\). Suppose in negation that \(p_j^* > 0\) for some such \(j\). Let \(p'\) be obtained from \(p^*\) by decreasing the \(j\)'s audit probability to \(p_j^* - \varepsilon\) and keeping all the other probabilities unchanged. Since \(j\) is not in the support of \(\beta^*\), \(p'\) and \(p^*\) yield the same expected payoff to the principal and since \(p^*\) is the best response against \(\beta^*\) so is \(p'\). For \(k > j\), \(A_{k,j}(p_j^*) < A_{k,s}(p_s^*)\), because \(p_s^* = p_s^* = 0\) and \(t_j > t_s\). Hence, for \(k > j\) agents' best response is to declare \(x_s\) with probability 1. This proves that \(\beta^*\) is a best response versus \(p'\). Thus, \((\beta^*, p')\) is an equilibrium, in contradiction to \((\beta^*, p^*)\) being a bureaucratically efficient equilibrium.

**Proof of Proposition 3**

Since our game is finite (i.e., there are finite number of states, finite number of players, and every player has a finite number of pure strategies), it admits at least one Nash equilibrium in behavioral strategies. Let \((\beta^*, p')\) be such an equilibrium. Define,

\[ K = \{p \in L : (\beta^*, p) \text{ is an equilibrium} \}, \]

where \(L\) denotes the set of all (behavioral) strategies of the principal. It is easily verified that \(K\) is a closed subset of \([0, 1]^N\), and therefore it is compact. Let \(p^* \in K\) be a principal's strategy at which the minimal value of \(\sum_{i=1}^{N} p_i\) is attained, then obviously \((\beta^*, p^*)\) is a bureaucratically efficient equilibrium. We proceed to show that every bureaucratically efficient equilibrium is sequential.

Fudenberg and Tirole proved that in two stage signaling games every perfect Bayesian equilibrium is a sequential equilibrium (see Fudenberg and Tirole [1991] ). Hence, in order to prove that a bureaucratically efficient equilibrium is a sequential equilibrium it suffices to prove that it is a perfect Bayesian equilibrium. A perfect Bayesian equilibrium is a Nash equilibrium \((\beta^*, p^*)\) which satisfies the additional property that for every \(i\) not in the support of \(\beta^*\) there exists a system of beliefs \(q(k|i), k = i, i + 1, \ldots, N\) (a distribution on all types \(k\), \(k \geq i\)) such that \(p_i^*\) maximizes the expected payoff to the principal given these beliefs. That is,

\[
\sum_{k=i}^{N} E_{k,i}(p_i^*)q(k|i) \geq \sum_{k=i}^{N} E_{k,i}(p_i)q(k|i) \quad \text{for all } p_i.
\]

Let \((\beta^*, p^*)\) be a bureaucratically efficient equilibrium and suppose that \(i \notin S(\beta^*)\). We have to construct beliefs for which (9.4) is satisfied. Namely, we have to establish beliefs \(q(i|i), q(i+1|i), \ldots, q(N|i)\), for which (9.4) is satisfied. To this effect we consider first the case \(p_i = 0\): Define beliefs according to which the principal believes with probability 1 that agents are truthful. That is, \(q(i|i) = 1\) and \(q(k|i) = 0\) for every \(i < k \leq N\). Obviously, such beliefs justify no audit; i.e. \(p*i = 0\). Next, consider the case where \(p_i > 0\). By (ii) in Proposition 2, \(p_N = 0\), and therefore \(i < N\). We construct now beliefs \(q(i|i), q(i+i|i), \ldots, q(N|i)\), which make the principal indifferent between auditing and not auditing after an announcement \(x_i\) is made. For such beliefs every \(p_i\) is optimal and in particular so is \(p_i^*\). The beliefs we suggest assign a positive probability only
to types $i$ and $N$. That is, $q(k|i) = 0$ for all $i < k < N$. Let $q(i|i) = d$ and $q(N|i) = 1 - d$.

Given these beliefs if the principal audits he collects $d(t_i - c_i) + (1 - d)(t_N + (t_N - t_i)\alpha - c_N)$, or, equivalently, $t_i + \delta$, 

$$
\delta = -c_id + [(t_N - t_i)(1 + \alpha) - c_N](1 - d). \quad (9.5)
$$

We wish to establish $d$ so that the principal is indifferent between auditing and not auditing and therefore we must have $\delta = 0$.

Since $c_i > 0$, and by (2.3) $t_N + f_{N,i} - c_N > t_i$, which is equivalent to $(t_N - t_i)(1 + \alpha) - c_N > 0$, such a constant $0 < d < 1$ exists.

References


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