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Theory and Methodology

# Hiring policies in an uncertain environment: Cost and productivity trade-offs

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**Abstract**

Firms naturally want to hire workers who are fast-learners and committed for long periods. Finding such people or carefully screening applicants is costly whether done directly by the firm or with the help of a recruitment agency. We explore the trade-offs involved in determining the effort/investment that should be expended on "quality control" in hiring. We envision a firm that has to decide which proportion of its hiring to do using a "careful" but expensive agency. Agencies may provide discounts to firms who do most of their hiring through them. Workers differ in their learning curve and in their (random) length of stay in the job/firm. We provide sufficient conditions for using only one agency. We also explore the relations between turnover and productivity, which at times turn out to be quite counter-intuitive. For example, reduced turnover may adversely effect productivity even in the presence of learning with experience. © 2000 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The negative impact of high labor turnover on firms in terms of productivity and hiring costs is widely recognized (e.g., Staw, 1980; Mobley, 1982; Cascio, 1987; Mercer, 1988; Phillips, 1990). McEvoy and Cascio (1985) analyze two strategies aimed at reducing turnover (realistic job previews and job enrichment programs) and find that the former strategy is generally more effective. Huselid (1995) study the impact of HRM practices on firm performance. Based on a survey of 1000 firms he argues that practices such as training, employee involvement, etc. cause significant turnover reduction and productivity enhancements. Pinkovitz et al. (1996) report on national turnover statistics in the USA and highlight the magnitude of turnover costs in different sectors. They develop a

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worksheet-based procedure to help calculate turnover costs. Fitz-Enz (1977) reports on findings gathered at the Saratoga Institute where the costs of turnover were investigated. Nevertheless, as stated by Hutchinson et al. (1997, pp. 3202–3203), "While there is an immense literature covering the subject of personnel turnover, there is a paucity of writing on the impact of turnover on the organization". Hutchinson et al. (1997) also point out that little attention has been paid to the effect of high turnover on throughput, via the impact of learning curves. It is our intention to explore the implication of hiring policies on turnover, productivity and output in an environment of uncertain turnover.

Partly as a result of the importance and difficulty of identifying potentially stable (i.e., committed to the firm for relatively long time) and productive workers, firms are increasingly relying on personnel agencies and recruitment specialists to supply them with the "right" kind of workers. These situations are encountered in various industries and with different functional positions. For example, an Israeli manufacturer of advanced medical diagnostic equipment experienced annual turnover of about 25% in its engineering workforce during the late 1980s. Management viewed this situation as not altogether negative since the company enjoyed a fresh supply of younger engineers who created an atmosphere of continuing rejuvenation. The company simultaneously employed several manpower agencies to meet its needs for qualified engineers. Hiring requirements were spread among the competing agencies and priority was given to agencies with proven track record. Another example taken from the Israeli marketplace relates to metal cutting – an industry that experiences shortages of welders, tool and die workers, operators of computer numerically controlled machines, etc. These firms employ several personnel agencies in order to broaden their reach (especially along geographic and ethnic dimensions) and in order to avoid being dependent on a single recruiting channel (a policy that may lead to higher commission fees for any given agency). In most cases, the hiring needs expressed by the firm are not accompanied by price differences among the agencies. However, in some instances, a firm may be willing to pay an extra fee to specific agencies, especially when its manpower requirements are relatively large and urgent. For example, large construction companies in Israel repeatedly face shortages in skilled construction workers. In recent years, they have adopted the practice of allocating a certain proportion of their hiring needs to personnel agencies that specialize in recruiting foreign workers (mainly from Romania, Turkey and Thailand). The construction companies assign these agencies specific recruitment targets while leaving a certain proportion of the hiring to be done locally in Israel. Moreover, since there are marked differences in the costs involved in recruiting and transporting workers from these countries, the prices paid to the relevant agencies differ.

The personnel recruitment industry, like most others, consists of agencies which vary by quality and price. Quality here is, of course, multidimensional, but some of its key dimensions are the learning rate, productivity, and stability of workers the agency supplies. Naturally, though, while careful recruiters may be able to find workers who *tend* to be more stable and/or more productive or fast learners than those found by less careful agencies, the *actual* performance will vary from worker to worker. An analysis of the economic trade-offs involved in hiring has to take these uncertainties into account. As we shall see, this uncertainty could give rise to some surprising observations and implications.

Research on personnel agencies and their use by employers confronted with turnover has proliferated in the last decade. Holzer (1987) develops an employer search model in which firms choose hiring procedures as well as production levels. The outcomes of their choices (expected vacancies, expected productivity, resources devoted to recruitment, etc.) are then evaluated. Scott (1994) describes basic characteristics of personnel agencies, the nature of the competition among them and the reasons that employers use them. Feldman et al. (1997) surveys 1312 personnel agencies and find that agencies that are older, larger and generate larger revenues tend to be more generalist in nature (serve more industries, functions and geographical regions). These agencies are typically paid on a retainer (rather than contingency) basis. The survey strongly motivates differential contractual agreements between an employer and the agencies that serve its needs.

Consider then a firm which has to decide what proportion, if any, of its hiring business to conduct through each of the several personnel agencies. Each agency's track record is summarized by the completed-length-of-service (CLS) distribution of the workers it supplies, and by their realized learning curves. The price per worker charged by each agency may depend, to some extent, on a precommittment by the firm as to the proportion of its hiring to be done through the agency.

We formulate a general renewal–reward model of these trade-offs. We then optimize it with respect to the proportion of workers hired via each of the two agencies. If agencies provide "quantity (proportion of client's business) discounts", it might be optimal to use a mix of agencies. In the absence of such discounts, only a single agency will be used, and the choice will be according to a rather interpretable criterion (Proposition 1). The choice criterion is then specialized to cases where agencies differ only in the productivity (but not stability) of workers they supply, and vice versa.

One specific way to model the "quality" of a personnel agency is by the proportion of "good" workers, in terms of stability and productivity, among those it provides. A convenient device to capture that is a *mixture*. Potential recruits are assumed to consist of two subgroups, one relatively more stable and productive than the other. A higher quality agency is then one which supplies a higher proportion of the more desirable type.

It is instructive to zero in on a firm's choice between CLS distributions. That will be relevant if the only discerniable difference between agencies is in the stability of workers they supply. However, it is even more directly relevant to a situation where the firm itself is contemplating an internal turnover-reduction effort (not necessarily through more careful hiring), in the hope of achieving productivity gains.

The rest of the paper is organized as follows. Section 2 presents the general renewal–reward model of the trade-offs involved. Section 3 focuses on the case where no quantity discounts are offered. Section 4 analyzes the effects of stability (reduced turnover) on overall productivity and provides examples where increased turnover is beneficial. Section 5 offers some conclusions.

## 2. The general model

Suppose that personnel agency $i, i = 1, \ldots, n$, charges the firm $h_i(\alpha_i)$ per recruit if the firm commits to hire a fraction $\alpha_i$ of its workers from that agency, where $\sum_{i=1}^{n} \alpha_i = 1$. We restrict the current model to situations in which the proportions $(\alpha_i)$ hired through various agencies remain fixed throughout the analysis horizon. Naturally, $h_i$ are nonincreasing functions. Internal expenses incurred by the firm in each turnover instance are also included in $h_i$. Agency $i$ delivers workers whose random length of service has a cumulative distribution function $F_i$, referred to as the CLS distribution (Bartholomew, 1982; Bartholomew et al., 1991). Let $\mu_i$ be the corresponding expected value of the completed length-of-stay of workers supplied by agency $i$.

To simplify the presentation and notation, in the rest of this section we focus on the case $n = 2$. Then, writing $\alpha_1 \equiv \alpha$ and $\alpha_2 = 1 - \alpha$, we have

$$E(\text{CLS}) = \alpha\mu_1 + (1 - \alpha)\mu_2. \quad (1)$$

Assume, without loss of generality, that the revenue per unit of output is 1. The workers supplied by agency $i$ exhibit a production/learning curve function $\phi_i$, where $\phi_i(x)$ is the quality-adjusted output per-unit-time of a worker with experience (i.e., length of service) $x$. Naturally, $\phi_i(x)$ are nondecreasing functions (e.g., Yelle, 1979; Smunt, 1986; Gerchak et al., 1990). In practice, initial productivity could vary by prior experience and learning rate could also vary from worker to worker. However, in order to highlight the consequences of uncertain length-of-stay we posit a deterministic production/learning function $\phi$.

If the successive CLSs are independent and identically distributed, a rather plausible assumption, which we shall make (in line with Bartholomew, 1982; Stanford, 1985 and others), the turnover in each position (job) can be modeled as a renewal process (Bartholomew, 1982; Stanford, 1985; Gerchak and Kubat, 1986). Thus, a worker's experience in a job at any point in time is the "age" of the renewal process at that time. The (asymptotic) age distribution of a renewal process equals (e.g., Ross, 1996; Wolff, 1989),

$$\int_0^x \bar{F}(y)\,\mathrm{d}y/\mu, \tag{2}$$

where $\bar{F} = 1 - F$. This distribution is called the "equilibrium" distribution $F_e$ corresponding to $F$. Its mean equals

$$\mu_e = E(X^2)/2\mu. \tag{3}$$

Thus, the expected productivity of a "random" worker is

$$\int_0^\infty \phi(x)\,\mathrm{d}F_e(x) = \int_0^\infty \phi(x)\bar{F}(x)\,\mathrm{d}x/\mu. \tag{4}$$

As workers originating from different agencies have different CLS distributions, their age distributions will also be different. Let

$$R_i \equiv \int_0^\infty \phi_i(x)\bar{F}_i(x)\,\mathrm{d}x/\mu_i, \quad i = 1, 2. \tag{5}$$

be the expected productivity of a random-experience worker originally supplied by agency $i$.

Regarding the system as a renewal–reward process, the long-term average profit per employee per unit time can be computed as the ratio of the expected profit per "cycle" (an employee's career at the job) and the expected length of cycle (given in Eq. (1)).

Denoting this ratio by $P(\alpha)$, we thus have

$$P(\alpha) = \frac{\alpha R_1 + (1-\alpha)R_2 - \alpha h_1(\alpha) - (1-\alpha)h_2(\alpha)}{\alpha\mu_1 + (1-\alpha)\mu_2}. \tag{6}$$

Note that we have redefined $h_2$ to be a (nondecreasing) function of $\alpha$, the proportion of business handled by the other (first) agency.

Differentiating (6) with respect to $\alpha$ and simplifying, we obtain

$$P'(\alpha) = \frac{[\alpha\mu_1 + (1-\alpha)\mu_2]\,[\alpha(h_2' - h_1') - h_2'] + \mu_1 h_2 - \mu_2 h_1 + \mu_2 R_1 - \mu_1 R_2}{[\alpha\mu_1 + (1-\alpha)\mu_2]^2}. \tag{7}$$

It is not easy to establish conditions which guarantee concavity of $P(\alpha)$ at this level of generality. For the special case of equal mean CLSs, $\mu_1 = \mu_2 \equiv \mu$, we can write

$$P'(\alpha) = \{(h_2' - h_1')\alpha - h_2' + h_2 - h_1 + R_1 - R_2\}/\mu, \tag{8}$$

so

$$P''(\alpha) = \{(h_2'' - h_1'')\alpha - h_2'' + 2(h_2' - h_1')\}/\mu. \tag{9}$$

Recall that $h_1' < 0$ and $h_2' > 0$. It is thus plausible to assume that $h_1'' > 0$ and $h_2'' < 0$. However, the sign of $P''(\alpha)$ is not clear. Equating $P'(\alpha)$ to zero results in the optimality condition

$$(h_2' - h_1')\alpha - h_2' = -(h_2 - h_1 - \Delta R), \tag{10}$$

where $\Delta R \equiv R_1 - R_2$.

**Example.** Let $h_1(\alpha) = h_1 e^{1-\alpha}$, and $h_2(\alpha) = h_2 e^{\alpha-1}$, $0 \leqslant \alpha \leqslant 1$ , where $0 < h_1 < h_2$. These functions are illustrated in Fig. 1.

Then

$$h_1'(\alpha) = -h_1 e^{1-\alpha}, \qquad h_2'(\alpha) = h_2 e^{\alpha-1}$$
$$h_1''(\alpha) = h_1 e^{1-\alpha}, \qquad h_2''(\alpha) = h_2 e^{\alpha-1}.$$

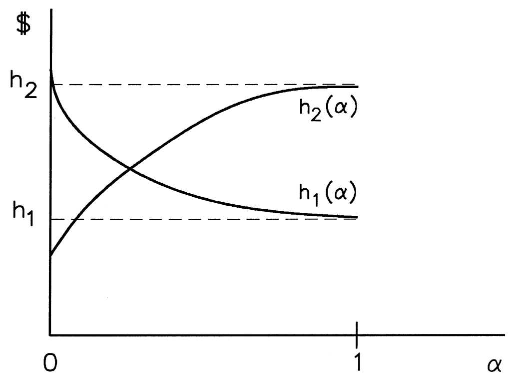


Fig. 1. Hiring cost functions.

Thus

$$\begin{aligned} P''(\alpha) &= \alpha\left(h_2 e^{\alpha-1} - h_1 e^{1-\alpha}\right) - h_2 e^{\alpha-1} \\ &\quad + 2\left(h_2 e^{\alpha-1} + h_1 e^{1-\alpha}\right) \\ &= (\alpha+1) h_2 e^{\alpha-1} + (2-\alpha) h_1 e^{1-\alpha} > 0, \\ &\quad \forall\ 0 \leqslant \alpha \leqslant 1 \\ &\Rightarrow \text{existence of a unique interior minimum.} \end{aligned}$$

Now, using (9), we equate $P'(\alpha)$ to zero,

$$\begin{aligned} P'(\alpha) = 0 &\Rightarrow \left(h_2 e^{\alpha-1} + h_1 e^{1-\alpha}\right)\alpha - h_2 e^{\alpha-1} + h_2 e^{\alpha-1} \\ &\quad - h_1 e^{1-\alpha} + R_1 - R_2 = 0 \\ &\Rightarrow \alpha h_2 e^{\alpha-1} + (\alpha-1) h_1 e^{1-\alpha} = R_2 - R_1. \end{aligned}$$

Thus if, for example, $h_2 = 2$, $h_1 = 1$ and $R_2 - R_1 = 0.8$, then the numerical solution of the above is $\alpha^* \approx 0.74$. That is, 74% of the hiring should be done via the first agency, which is cheaper but whose workers have a somewhat lower expected productivity.

## 3. No volume discounts

The prevailing practice in the personnel industry today does not provide firms with volume discounts. If agencies charge a fixed rate per hiree regardless of how much hiring the firm does through them, the functions $h_1$ and $h_2$ become constants. Then the expected profit per unit time becomes

$$P(\alpha) = \frac{\alpha R_1 + (1-\alpha) R_2 - \alpha h_1 - (1-\alpha) h_2}{\alpha \mu_1 + (1-\alpha)\mu_2}. \tag{11}$$

Thus,

$$P'(\alpha) = \frac{\mu_2 R_1 - \mu_1 R_2 - \mu_2 h_1 + \mu_1 h_2}{[\alpha\mu_1 + (1-\alpha)\mu_2]^2}. \tag{12}$$

We note that the numerator of $P'(\alpha)$ does not contain $\alpha$. Thus, $P(\alpha)$ is monotone, and its direction, which determines whether $\alpha = 0$ or $\alpha = 1$ is optimal, depends on the sign of the numerator of $P'(\alpha)$. Recalling what the $R_i$'s represent, we thus have:

**Proposition 1.** *Suppose that $h_1$ and $h_2$ are constant. Then if*

$$\frac{\int_0^\infty \phi_1(x)\bar{F}_1(x)\,dx - h_1}{\mu_1} \geqslant \frac{\int_0^\infty \phi_2(x)\bar{F}_2(x)\,dx - h_2}{\mu_2}, \tag{13}$$

$\alpha^* = 1$. *Otherwise*, $\alpha^* = 0$.

Since $\int_0^\infty \phi_i(x)\bar{F}_i(x)\,dx/\mu_i$ is the average revenue, and $h_i/\mu_i$ the average hiring cost per employee (per unit time), condition (13) states that the expected net profit (per unit time) of agency 1's workers is higher than of agency 2's workers, and thus the result is quite intuitive.

This result actually generalizes to any number of agencies: The only agency used will be the one with the highest value of

$$\frac{\int_0^\infty \phi_i(x)\bar{F}_i(x)\,dx - h_i}{\mu_i}. \tag{14}$$

Several important special cases are worth considering. The first will assume that the agencies supply workers with *equal mean length-of-stay* (as was the case in the previous Section's example). The variability of CLS and the productivity of workers supplied by different agencies may vary, however.

**Corollary 1.** *Suppose that $\mu_1 = \mu_2$. If*

$$\int_0^\infty \phi_1(x)\bar{F}_1(x)\,dx - \int_0^\infty \phi_2(x)\bar{F}_2(x)\,dx \geqslant h_1 - h_2 \tag{15}$$

*then* $\alpha^* = 1$. *Otherwise*, $\alpha^* = 0$. *If, in addition*:

(i) $F_1 = F_2 \equiv F$, *then the condition reduces to*

$$\int_0^\infty [\phi_1(x) - \phi_2(x)]\bar{F}(x)\,dx \geqslant h_1 - h_2; \tag{16}$$

(ii) $\phi_1 = \phi_2 \equiv \phi$ (*but* $F_1 \neq F_2$) *the condition becomes*

$$\int_0^\infty \phi(x)[\bar{F}_1(x) - \bar{F}_2(x)] \geqslant h_1 - h_2. \tag{17}$$

(iii) $h_1 = h_2$ (*but* $\phi_1 \neq \phi_2$ *and* $F_1 \neq F_2$) *the condition becomes*

$$\int_0^\infty \phi_1(x)\bar{F}_1(x)\,\mathrm{d}x \geqslant \int_0^\infty \phi_2(x)\bar{F}_2(x)\,\mathrm{d}x. \qquad \square \tag{18}$$

Conditions (16) and (17) state that if the difference in fees between the agencies is smaller than, respectively, the CLS-weighted "productivity difference" and the productivity-weighted "CLS-difference" the more expensive agency should be chosen. Condition (18) states that only the agency whose workers' expected productivity is higher will be used (if mean CLS and hiring costs are the same).

The next type of specialization assumes that $h_1/\mu_1 = h_2/\mu_2$ (though $\mu_1 \neq \mu_2$). That is, the price charged is proportional to the average stability of workers the agency supplies.

**Corollary 2.** *Suppose that* $h_1/\mu_1 = h_2/\mu_2$. *If*

$$\int_0^\infty \phi_1(x)\bar{F}_1(x)\,\mathrm{d}x/\mu_1 \geqslant \int_0^\infty \phi_2(x)\bar{F}_2(x)\,\mathrm{d}x/\mu_2 \tag{19}$$

*then* $\alpha^* = 1$. *Otherwise*, $\alpha^* = 0$. $\square$

That is, we use (only) the agency whose workers' expected productivity is higher. That agrees with intuition.

What if the only difference between the agencies of Corollary 2 is in the learning curves? That is, what if $F_1 = F_2 \equiv F$ (and hence, $\mu_1 = \mu_2$)? In that case, clearly, $\alpha^* = 1$ iff

$$\int_0^\infty [\phi_1(x) - \phi_2(x)]\bar{F}(x)\,\mathrm{d}x \geqslant 0. \tag{20}$$

(This property is also a special case of (i) in Corollary 1.) Thus, if $\phi_1(x) \geqslant \phi_2(x) \quad \forall x$, then $\alpha^* = 1$. That is, if the productivity of workers supplied by one agency is higher than that of the other agency at all experience levels, that agency should be the (only) one used. If such productivity-dominance does not hold, the choice will depend on $F$, as well as on $\phi_1$ and $\phi_2$.

The other relevant special case of (19) is $\phi_1 = \phi_2$ (but $F_1 \neq F_2$ and, possibly, $\mu_1 \neq \mu_2$). Here the agencies differ only in the stability of the workers they send. The initial productivity and learning rate do not vary by agency, but learning does take place. Thus we are comparing the quantities

$$\int_0^\infty \phi(x)\bar{F}_1(x)\,\mathrm{d}x/\mu_1 \quad \text{vs.} \quad \int_0^\infty \phi(x)\bar{F}_2(x)\,\mathrm{d}x/\mu_2.$$

This scenario is, in fact, quite rich, and can give rise to a variety of behaviors, some quite surprising. We shall analyze it in some detail in Section 4, under the assumption that one agency supplies workers which tend to be more "stable".

## 4. Comparing stability patterns

What does one actually mean when saying that some type of workers are more "stable" than others? Presumably this is intended to say that one type's CLS is longer than the other's. The CLS's being random, however, the exact meaning of "longer" is not obvious. The natural concepts of ordering random variables (or, equivalently, their distributions) are those of *stochastic order* (e.g., Ross, 1996; Shaked and Shanthikumar, 1994). We shall not provide here a complete review of these concepts, but only define those which will be used in our context.

Let $X$ and $Y$ be random variables with distributions $F$ and $G$ respectively, and let $\bar{F} = 1 - F$, $\bar{G} = 1 - G$.

If $\bar{F}(t)/\bar{G}(t)$ decreases in $t$, $X$ is said to be smaller than $Y$ in the *hazard rate order*, denoted by $X \leqslant_{\mathrm{hr}} Y$.

If $\bar{F}(t) \leqslant \bar{G}(t)\ \forall t$, $X$ is said to be smaller than $Y$ in the usual *stochastic order*, denoted by $X \leqslant_{\mathrm{st}} Y$. This can be shown to be equivalent to $E[\phi(X)] \leqslant E[\phi(Y)]$ for all increasing functions $\phi$.

If $\int_x^\infty \bar{F}(t)\,\mathrm{d}t \leqslant \int_x^\infty \bar{G}(t)\,\mathrm{d}t\ \forall x$, $X$ is said to be smaller than $Y$ in the *increasing convex ("variability") order*, denoted by $X \leqslant_{\mathrm{icx}} Y$. This can be shown to be equivalent to $E[\phi(X)] \leqslant E[\phi(Y)]$ for all increasing convex functions $\phi$.

Clearly

$$X \leqslant_{\mathrm{hr}} Y \Rightarrow X \leqslant_{\mathrm{st}} Y \Rightarrow X \leqslant_{\mathrm{icx}} Y \Rightarrow E(X) \leqslant E(Y).$$

If the distributions of $X$ and $Y$ are not identical, then $X \leqslant_{\mathrm{st}} Y \Rightarrow E(X) < E(Y)$, but the combination

$X \leqslant_{\rm icx} Y$ and $E(X) = E(Y)$ is possible. Indeed, in the case of nonnegative variables with equal means, $X \leqslant_{\rm icx} Y \iff X \leqslant_{\rm cx} Y$, where the latter order is defined as $E[\phi(X)] \leqslant E[\phi(Y)]$ for all convex functions $\phi$ (and thus, in particular, for variances).

Let $X_{\rm e}$ and $Y_{\rm e}$ be the equilibrium variables corresponding to $X$ and $Y$ respectively. (Eq. (2) gives the relation between equilibrium and original distribution.) Now, we know from (4) that the expected productivity of a worker with CLS $X \sim F$ equals

$$E[\phi(X_{\rm e})] \equiv \int_0^\infty \phi(x)\,{\rm d}F_{\rm e}(x) = \int_0^\infty \phi(x)\bar{F}(x)\,{\rm d}x/\mu \ .$$

Thus, *if* $X_{\rm e} \leqslant_{\rm st} Y_{\rm e}$ then the expected output associated with CLS $Y$ will be larger than with CLS $X$ for *all* productivity functions $\phi$.

The question is, then, what type of order on $X$ and $Y$ induces $X_{\rm e} \leqslant_{\rm st} Y_{\rm e}$. In the case of *equal means*, the answer is rather simple:

**Proposition 2** [Wolff, 1989, p. 488]. *If* $E(X) = E(Y)$ *then*

$$X \leqslant_{\rm icx} Y \iff X_{\rm e} \leqslant_{\rm st} Y_{\rm e}.$$

Thus, an increase in the variability of the CLS, without change in mean, always increases the expected output. Increased uncertainty is thus beneficial here (!). The intuitive explanation of this is that an increase in CLS' variability, by causing the long-stayers to stay even longer, ups productivity here more than the negative contribution to productivity caused by the short-stayer staying even shorter times. See also example given later.

Once the equal-means assumption is removed, the situation changes drastically. In particular, Wolff (p. 489) provides an example which shows that it is possible for both $X \leqslant_{\rm st} Y$ and $X_{\rm e} \geqslant_{\rm st} Y_{\rm e}$ to hold simultaneously in the strict sense. Thus, it is possible that expected productivity will go *down* for *any* increasing learning curve even though the new CLS is stochastically longer (!)

Nevertheless, even when means are not equal, sufficient strengthening of the $X \leqslant Y$ order *does* imply that $X_{\rm e}$ and $Y_{\rm e}$ will be ordered.

**Proposition 3** [Whitt, 1985].

$$X \leqslant_{\rm hr} Y \Rightarrow X_{\rm e} \leqslant_{\rm st} Y_{\rm e}.$$

Thus if scenario $Y$'s stability is higher (turnover is lower) than scenario $X$'s in the sense of hazard rate order (which is quite strong), experience will increase stochastically and thus will productivity.

To gain more insight into which types of turnover reductions (internal, or via switching personnel agencies) are guaranteed to improve productivity and which do not, suppose that the CLS before the change has the simple two-point distribution

$$X = \begin{cases} a & \text{w.p. } p, \\ b & \text{w.p. } 1-p, \end{cases}$$

where $b > a > 0$. The manpower planning literature refers to the group that stays a shorter time as "fast movers" and to those who stay longer "slow movers"" (e.g., Bartholomew et al., 1991). Starting with this base-scenario, we observe the outcomes of potential changes in the parameters of the CLS distribution (this example is based on Gerchak and He, 1995):

(a) Suppose the new situation is

$$Y = \begin{cases} a-\epsilon, & \text{w.p. } p, \\ b+\epsilon, & \text{w.p. } 1-p, \end{cases}$$

i.e., the fast movers leave even faster, while the slow movers stay longer. If $p = 0.5$, in which case the mean stays unchanged, the expected productivity will go up. (Proposition 2). No surprise here.

(b) Suppose the new situation is

$$Y = \begin{cases} a+\epsilon, & \text{w.p. } p, \\ b, & \text{w.p. } 1-p, \end{cases} \quad \epsilon > 0,$$

i.e., the fast movers stay longer than before. Then the productivity might not go up. For example, if $p = 0.5$, $a = \epsilon = 1$ and $b = 4$, then $E(X_{\rm e}) = 1.7$, while $E(Y_{\rm e}) = 5/3$, and thus if $\phi(x) = x$ (in which case expected productivity simply equals expected age), expected productivity will go *down*. Indeed, expected productivity will go down iff $\epsilon < 1.4$.

An intuitive explanation to the seemingly "strange" behavior in this example is as follows. In the base ($X$) scenario, slow movers tend to occupy the "job" 4/5 of the time, while in the "lower turnover" scenario ($Y$) they occupy it only 4/6 of the time. Thus, in the base scenario a "random" worker is more likely to be a slow mover and hence, on the average, a more experienced worker.

We note that $E(Y_e) > E(X_e)$ guarantees an increase in expected productivity only if productivity is linear in age. Improvement is guaranteed for *any* productivity curve only when $Y_e \geqslant_{st} X_e$ holds. However, in our example,

$$F_e(x) = \begin{cases} 2x/(5+\epsilon), & x \leqslant 1+\epsilon, \\ (x+\epsilon+1)/(5+\epsilon), & 1+\epsilon < x \leqslant 4, \end{cases}$$

which, in the latter range, is *increasing* in $\epsilon$. Thus, this type of turnover reduction cannot guarantee an increase in productivity.

(c) Suppose the new situation is

$$Y = \begin{cases} a, & \text{w.p. } p-\delta, \\ b, & \text{w.p. } 1-p+\delta, \end{cases} \quad \delta > 0,$$

i.e., the fraction of slow movers went up. Here $X \leqslant_{hr} Y$. Then, despite the change in mean CLS, the expected productivity will always go up (Proposition 3).

## 5. Concluding remarks

This paper analyzes the trade-offs involved in hiring workers, primarily through external agencies, aiming at balancing hiring cost and expected contribution by workers, while accounting for their stability and learning capabilities. We show that when the agencies that supply the workers offer incentives in exchange for increased share of the hiring business, the optimal policy for the firm involved may well be to use a *mix* of several agencies for its personnel needs. Although we do not show it explicitly, this outcome may be akin to "second sourcing" policies in inventory replenishment. That is, even if no *explicit* incentives were offered by the agencies, a more appropriate model for describing the economics of hiring should include *implicit* charges to express the potential risk that an agency will not be able to supply all the workers it normally does. Adding such implicit costs would lead to mixed policies, similar to the consequences of the nonlinear costs here.

In the case of constant hiring charges, we show that firms will always prefer a single agency, and we provide the rules for determining the preference. We note, however, that this solution is not likely to be stable in dynamic settings. Other manpower agencies not selected by the firm in the first period will look for ways to change the situation in subsequent periods by reducing their fees or by offering a different mix of workers. Finding an equilibrium for such an environment could possibly be achieved through game-theoretic models, but since this goes beyond the scope of this paper, we did not pursue it.

Since in practice many firms are functionally decentralized, the personnel department may only have a limited hiring budget. In the case that a single agency (say, #1) ought to be used, the expected hiring costs per period are $h_1/\mu_1$. If that exceed the hiring budget, while $h_2/\mu_2$ does not, a mix of agencies may need to be used. The implications of budget constraints require further research.

The most intriguing results of this paper are given in Section 4. There we present counter-intuitive scenarios in which increased turnover actually improves productivity. The importance of this surprising result goes beyond manpower hiring.

The models developed in this paper are applicable in various industries and to different manpower functions. For the most part, we address situations in which a firm is recruiting salaried employees to positions that are characterized by relatively high turnover. Possible application areas cover both manufacturing and service industries. In manufacturing, these models may apply to the hiring of production workers with specific skills (welders, tool and die workers, machinists), assembly workers or engineers at entry level positions in the high-tech manufacturing sector, skilled construction workers, etc. In service industries, the models could be relevant to the hiring of bank tellers, supermarket cashiers, cooks and skilled kitchen workers for hospitals and other public facilities, etc.

On the other hand, the framework offered by our models does not fit the hiring of top executives, which is typically done by head-hunters operating in circumstances different than those assumed here (see also Terborg and Lee, 1984). It is also unsuitable for modeling the hiring of low-level temporary manpower that is normally reached through classified ads or through government employment agencies. Other marketplace scenarios not captured by the model include cases where personnel agencies represent the employees and, on the other extreme, cases where an in-house personnel office assumes full responsibility over hiring procedures. Outsourcing is another marketplace trend that is not discussed here. Our experience in Israel shows that a growing number of firms are outsourcing activities that are characterized by large turnover. For example, commercial banks that used to hire all of their tellers as salaried employees (due to both conservative management practices and powerful union agreements) have recently outsourced a growing proportion of the teller activities to manpower agencies. Exploring turnover/productivity/cost trade-offs in such professions and institutional settings would be a worthwhile future research direction.

The practice of making a conscious decision to assign specific proportions of a firm's hiring needs to particular agencies is presently not wide-spread among employers. However, there are cases in which these phenomena do occur. The construction industry example, discussed in the Introduction, illustrates real-world situations where the model's assumptions hold. Another example concerns a large manufacturer who recently decided to open a production facility where hundreds of assembly workers will be required through the lifetime of the plant. The manufacturer is ready to commit a certain proportion of its hiring business to an agency that will guarantee supply of qualified workers whenever needed. The agency, on the other hand, is willing to negotiate a special (reduced) price per hiree to secure this lucrative business for many years.

Finally, we wish to stress that the model here should be viewed primarily as normative rather than empirical. Even though it addresses practices that are rather rare today, it might provide the tools for a variety of employers and personnel agencies to enter into future business relations in ways that will be more beneficial to both parties.

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