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Constructing and evaluating balanced portfolios of R&D projects with interactions: A DEA based methodology

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Abstract

We propose and demonstrate a methodology for the construction and analysis of efficient, effective and balanced portfolios of R&D projects with interactions. The methodology is based on an extended data envelopment analysis (DEA) model that quantifies some the qualitative concepts embedded in the balanced scorecard (BSC) approach. The methodology includes a resource allocation scheme, an evaluation of individual projects, screening of projects based on their relative values and on portfolio requirements, and finally a construction and evaluation of portfolios. The DEA–BSC model is employed in two versions, first to evaluate individual R&D projects, and then to evaluate alternative R&D portfolios. To generate portfolio alternatives, we apply a branch-and-bound algorithm, and use an accumulation function that accounts for possible interactions among projects. The entire methodology is illustrated via an example in the context of a governmental agency charged with selecting technological projects.

Keywords: Data envelopment analysis; R&D projects; Portfolio analysis; Balanced scorecard

1. Introduction

Portfolio selection problems can be decomposed into two major classes: dynamic vs. static problems. In the dynamic class (Bard et al., 1988; Cooper et al., 1997), at every decision point there are projects that have already started—denoted as *active* projects, and a set of proposed projects—known as *candidate* projects. The decision space includes both groups, and may involve the continuation of active projects at various budgeting levels; termination of other active projects; and launching new projects. In this paper we focus

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Nomenclature

n_p	total number of projects
G	a group of candidate projects
Q_k	the group of projects in portfolio k
m	number of inputs (resources) for a project/portfolio
s	number of outputs (benefit dimensions) for a project/portfolio
R	$m \times n_p$ matrix of available/allocated inputs (resources)
R_i	total amount of input/resource i available
r_{ic}	amount of input (resource) i allocated to category c
x_{ij}	amount of input (resource) i required for project j
y_{rj}	amount of output (benefit) r expected from project j given success
v, μ	vectors of variables associated with the inputs and outputs in the DEA model
$\hat{x}_{ik}$	amount of input (resource) i required for portfolio k
$\hat{y}_{rk}$	amount of output (benefit) r expected from portfolio k given success
p_j	probability of success of project j
U^i	the resource interaction matrix of input (resource) i
V^r	the value interaction matrix of output (benefit) r
P	the outcome interaction matrix
B^r	the expected value interaction matrix of output (benefit) r
z_k	a vector that represents a particular selection of projects in portfolio k ($z_{jk} = 1$ if project j is included portfolio k, otherwise $z_{jk} = 0$)
C_ℓ	group ℓ of measures representing a BSC-like card

on the class of static portfolio selection problems (e.g., Beaujon et al., 2001; Basso and Peccati, 2001). This class addresses situations in which all the projects that are considered at the decision point are candidates.

The static setting may occur in both the business and the government sectors. As an example of the former, consider a venture capital firm that wishes to invest resources in a set of new technologies. It sets aside a certain budget dedicated for this purpose and announces a "call-for-proposals" to solicit proposals in various areas. Similarly, in the not-for-profit sector, a governmental agency may have a certain budget dedicated for new projects. Decision points may occur once a period, and the decision is which new projects to support.

Such decision problems are an important management issue (Roussel et al., 1991; Cooper et al., 1997). Given that in technology-based organizations the technology strategy is directly linked to the organization's strategy, medium and long-term success of such organizations is often determined by the effectiveness of the portfolio selection process (Roussel et al., 1991, Chapter 6; Schmidt and Freeland, 1992).

The rationale of constructing an R&D project portfolio is quite similar to that of constructing a financial portfolio. As observed by Markowitz (2002), an investing agent concerned only with the expected value of financial options would have been required to invest only in one stock to maximize the value of such a portfolio. However, diversification of financial options is a common practice whose aim is to avoid the risk of "putting all eggs in one basket". The same rationale works in the context of R&D projects where risk is one of the main characteristics of the environment and, typically, a single R&D project cannot reflect properly the many objectives of an R&D strategy. Hence, diversification in R&D projects is essential.

The static portfolio selection problem is a complex one. It is concerned with the allocation of scarce resources, such as funds, manpower and facilities, to a set of candidate projects that best serves the objectives of the relevant organization, in the face of tradeoffs among key strategic dimensions (e.g. risk and reward, stability and growth, short-term and long-term benefits). The performance evaluation is not limited to profitability, and it usually requires the consideration of multiple criteria, many of which involve uncertain and/or subjective data. Even for-profit organizations would typically view the "value" of a project or a portfolio as a vector of several components including qualitative and subjective measures such as: the extent to which the selection establishes a platform for growth, the projects complexity, and the competitive intensity. In not-for-profit organizations, the role played by qualitative and subjective measures becomes even more dominant, and performance is usually measured by several incomparable outputs, with no accepted way of combining them into a single number that measures the overall effectiveness (Anthony and Herzlinger, 1980, Chapter 2). While quantitative measures are rarely forecasted accurately and are usually bound to considerable uncertainty, qualitative factors may dominate the R&D performance evaluation. The selection and relative importance of specific criteria may also differ according to the strategy and objectives of the sponsoring organization, as well as on the nature of the R&D activity. Hence, methods that seek to support the R&D portfolio analysis process must accommodate subjective judgment and uncertain data, and adapt to the specifics of the organization and its environment.

1.1. Existing methods for the static portfolio selection problem

Many theoretical and practical attempts have been made to develop models that would support the process of portfolio selection. Early attempts focused on theoretical operations research and management science models, usually in the form of a constrained optimization problem. Given a set of candidate projects, the goal is to select a subset of projects to maximize some objective function without violating the constraints (Baker, 1974; Liberatore and Titus, 1983; Liberatore, 1988; Danila, 1989). However, these models have not found widespread use in practice. This phenomenon was observed by Hall and Nauda (1990) who noted that these models require accurate data that is unavailable in most cases. The same phenomenon was also observed by Schmidt and Freeland (1992) and later confirmed by Cooper (2001, Chapter 8). Farrukh et al. (2000) offer possible explanations for the limited implementation of such models in practice (Farrukh et al., 2000, p. 45) and Loch et al. (2001) describe similar experience in a real-world setting.

More recent models try to close some of the gaps that were observed earlier. For example, the expected commercial value (ECV) (Cooper et al., 1997, Chapter 2) is based on a decision tree and incorporates estimates of probability of technical success and commercial success. Henriksen and Traynor (1999) presented an improved scoring method that explicitly incorporates tradeoffs among the evaluation criteria and calculates a relative measure of project value by taking into account the fact that value is a function of both merit and cost. Beaujon et al. (2001) presented a mixed integer-programming model in the form of a multi-dimensional knapsack problem, and introduced balance targets by incorporating constraints that are related to the inputs (specifically, the budget). However, quantitative models in portfolio selection still capture only part of the picture, and they are often deemed as incomplete.

Baker and Freeland (1975) emphasized a drawback of many selection models. In their review of existing methods at that time, they concluded that "one of the most important limitations of present R&D project selection models is the inadequate treatment of project interrelationships with respect to both value and resource utilization." Not much has changed since then. The problem of interactions among projects received relatively little attention in the literature. Schmidt (1993) proposed a model that accounts for the combined effect of resources, benefits, and outcome interactions. He used a nonlinear integer program for resource allocation, and proposed a branch-and-bound algorithm to solve it. Dickinson et al. (2001) demonstrated the use of a dependency matrix, which quantifies benefit interactions in a portfolio optimization model that was developed for the Boeing Company. Verma and Sinha (2002) developed a theoretical framework, through multiple case studies of projects undertaken by high tech manufacturing firms, for understanding the interdependencies between projects and their relationship to project performance in multi-project R&D environments.

In spite of the many different models that have been proposed, practitioners still consider this problem to be unsolved. Thus, there is a strong motivation for further studies of this problem.

1.2. A combined data envelopment analysis (DEA) and balanced scorecard (BSC) approach for portfolio selection

While the portfolio management methods that are employed in various organizations vary greatly, the objectives that managers are trying to achieve are quite similar. Cooper et al. (1997) recognized three broad objectives that usually dominate this decision process:

Effectiveness: The alignment of the mix of projects in the portfolio with the strategic goals of the organization.

Efficiency: The value of the portfolio in terms of long-term profitability, return-on-investment, likelihood of success, or other relevant performance measures.

Balance: The diversification of the projects in the portfolio in terms of various tradeoffs such as high risk versus sure bets, internal versus outsourced work, even distribution across industries, etc.

In this paper we propose a new methodology that responds to all three major objectives. The methodology employs a model based on data envelopment analysis (DEA) and balanced scorecard (BSC). DEA was developed by Charnes et al. (1978) to evaluate the relative efficiency of decision making units (DMUs). DEA handles DMUs that are engaged in performing similar functions using a set of inputs to produce a set of outputs. Both the inputs and the outputs may contain quantitative as well as qualitative factors. The basic DEA model defines efficiency, as the ratio of the weighted sum of outputs to the weighted sum of inputs. The model chooses for each DMU the set of weights that achieves the highest efficiency rating, while assuring that these weights do not cause any other DMU to have an efficiency rating higher that 1.0. The usefulness of DEA in evaluating multi-criteria systems and providing improvement targets for such systems gave birth to numerous applications that extended its use from the traditional efficiency studies to various decision-making problems. A comprehensive reference list on DEA and its applications can be found in Cooper et al. (2000). Some DEA applications have emerged in recent years in the context of projects' evaluation. These applications usually use adapted DEA models with better discriminating power, that are capable of ranking alternatives beyond the dichotomized classification of DMU into efficient and inefficient groups. For example, Oral et al. (1991) presented a methodology for evaluating and selecting R&D projects in a collective decision setting. They used DEA to determine the relative value of a given R&D project from the viewpoint of the other R&D projects. Linton et al. (2002) proposed a method for the analysis, ranking, and selection of R&D projects using the basic DEA model to split a set of projects into "accept", "consider further", and "reject" sub-groups, and subsequently applied graphic methods for the portfolio analysis.

BSC is a concept that was presented by Kaplan and Norton (1992, 1996a,b) as an organizational measurement system. This methodology was motivated by the realization that traditional financial accounting measures, such as return-on-investment, may be incomplete and, if used alone, may yield misleading signals for continuous improvement and growth of organizations. BSC is aimed at producing a balanced representation of the firm's performance. It does so by focusing on four groups of performance measures (denoted by Kaplan and Norton as "cards"): *financial*, *market*, *internal growth* and *innovation*. For each card, the method identifies a number of measures that provide a comprehensive evaluation of the organization's performance in the dimensions of its operations that correspond to the specific card. BSC has some advantages: it minimizes information overload by limiting the number of measures it uses; it brings together many seemingly disparate elements of the evaluation; and finally, it guards against sub-optimization by considering all the important measures together, while providing the ability to see whether improvement in one area may have been achieved at the expense of another.

In Eilat et al. (2004), we proposed a DEA–BSC model that incorporates the BSC concept into the mathematical programming formulation of DEA by introducing additional constraints that limit the feasible values of the weights in the original model. While DEA models with restrictions on weights have already been developed and tested (see the earlier work by Charnes et al. (1989), Thompson and Langemeier (1990), Roll et al. (1991) and the more recent review in Allen et al. (1997)), the novelty of our DEA–BSC modeling approach is twofold. First, from the BSC perspective, we offer a way to quantify the BSC concept. Second, from the DEA perspective, we establish a hierarchical structure (corresponding to the BSC cards) of weight restrictions in DEA.

In this paper, we extend the work of Eilat et al. (2004) from a single to a multiple-project environment. Our main contribution in this paper is the development of a quantitative methodology for the analysis of static portfolio selection problems that responds to the three goals of effectiveness, efficiency, and balance, and takes into account combined benefit, outcome and resource interactions. The methodology distinguishes considerations taken at the project level (e.g. individual project efficiency) from those taken at the portfolio level (e.g. balance of risk and reward within the portfolio). It also extends the traditional analysis of balance considerations and considers balance among inputs (i.e., balance of resources allocated to different categories of projects) as well as balance among outputs (i.e., balance among the value of the benefits obtained in different strategic dimensions). The method employs a portfolio generation algorithm and an accumulation function that takes into account possible complex interactions among projects. It then applies the DEA–BSC model to evaluate the alternative portfolios and select the best one(s).

The remainder of the paper is organized as follows. Section 2 presents the DEA–BSC model. Section 3 presents the new methodology for R&D portfolio selection that is based on the DEA–BSC as an evaluation mechanism. In Section 4 we use a numerical example to demonstrate the methodology. Concluding remarks are given in the last section.

2. The DEA–BSC model

DEA is a mathematical programming technique that calculates the relative efficiency of multiple decision-making units (DMUs) based on multiple inputs and outputs. Assume n DMUs, each consuming m inputs and producing s outputs. In our context, we regard these DMUs as either projects or portfolios. Let the vector $X_j = \{x_{ij}\}$ be the observed inputs ($i = 1, \ldots, m$) and the vector $Y_j = \{y_{rj}\}$ be the observed outputs ($r = 1, \ldots, s$) of project j ($j = 1, \ldots, n$). The relative value of a specific DMU, A_o, is defined in the basic DEA model (the CCR model given by Charnes et al., 1978) as the ratio between the weighted sum of outputs to the weighted sum of inputs. The weights are the variables of the model. They are defined in a way that allows each DMU in turn present itself in the most favorable way. Formulation (1) presents the CCR model in its ratio form. The constant ε is an infinitesimal number that functions as a lower bound for the weights.

$$\begin{aligned}
\max_{u,v} \quad & \frac{\sum_r \mu_r y_{ro}}{\sum_i v_i x_{io}} \\
\text{s.t.} \quad & \frac{\sum_i \mu_i y_{ij}}{\sum_i v_i x_{ij}} \leqslant 1 \quad \forall j, \\
& \mu_r \geqslant \varepsilon, \\
& v_i \geqslant \varepsilon.
\end{aligned} \tag{1}$$

By solving model (1) n times (each time evaluating a different DMU at the objective function), we get relative efficiency scores for all the DMUs. These scores assign the DMUs into two groups: the 'efficient' ones (with scores of 1.0) lying on the 'efficiency frontier' or 'envelope', and the inefficient ones (with scores smaller than 1.0) that fall below the frontier.

To construct a DEA–BSC model, one needs to determine, for each area of application (in our case, selection of R&D projects), an appropriate set of BSC-like cards that classify the inputs and outputs into cards where each card represents a major dimension of interest for the multi-project organization. Then, the appropriate measures within each card should be defined. To allow multiple-level balance restrictions, we treat the general structure of outputs or inputs in the DEA–BSC model as a hierarchy. In what follows we will demonstrate the hierarchical structure of the outputs (the concept is also true for the corresponding structure of inputs). The highest hierarchical level includes a single card, denoted by C_0, which includes all the output measures. The next level includes the cards $C_1, \ldots, C_{\ell_0}$, that represent the first partition level of C_0 into ℓ_0 cards. Each card in the second level can be broken down into sub-card and the process is thus repeated until, at the lowest level of the hierarchy we find the output measures themselves.

To introduce the balance restrictions, we refer to the first partition level.

Since by definition,

$$\sum_{\ell=1}^{\ell_0}\left(\sum_{i\in C_\ell}\mu_i y_{ij}\Big/\sum_{i\in C_0}\mu_i y_{ij}\right)=1,\quad \forall j, \tag{2}$$

we have that each component in the first summation in (2) represents the proportion of the total output of DMU A_j devoted to card C_ℓ. We regard this component as the 'importance' attached to card C_ℓ by DMU A_j. The larger this expression, the more A_j depends upon outputs in C_ℓ in determining its score. To reflect the desired balance, a decision-maker should set limits using suitable lower and upper bounds on the relative importance in each card. Formally, the restrictions presented in inequalities (3) are imposed on any specific alternative A_o that is being evaluated.

$$L_{C_\ell}\leqslant \sum_{i\in C_\ell}\mu_i y_{io}\Big/\sum_{i\in C_0}\mu_i y_{io}\leqslant U_{C_\ell},\quad \forall \ell. \tag{3}$$

3. The proposed methodology

The proposed methodology is composed of seven steps. It begins with a resource allocation scheme (Step 1) that distributes resources (inputs, in the DEA terminology) among categories related to key strategic dimensions (e.g. product lines, technological areas, strategic goals, project types). After resources are distributed, R&D projects within each category are modeled as DMUs, and evaluated with the DEA–BSC model (Step 2). The efficiency scores of the individual projects are used to split the projects into 'consider further' (i.e., candidate list) and 'reject' sub-groups. Project indices are then computed to allow control over the variability of the risk, efficiency, and balance of outputs (Step 3). A branch-and-bound model is then applied to generate alternative portfolios for subsequent evaluation (Step 4). Then, an accumulation function that takes into account the combined effect of possible benefit, outcome, and resource interactions is applied to the inputs and outputs of the projects in each portfolio to determine aggregate portfolio inputs and outputs (Step 5). To evaluate the alternative portfolios, the DEA–BSC model is used again, this time at the portfolio level (Step 6). Finally, sensitivity analysis is performed, and a desirable portfolio is selected (Step 7). In what follows we describe the methodology in details.

3.1. Step 1: Allocation of resources

The first step of the portfolio selection process allocates resources (inputs) to projects. We follow an iterative top–down approach that is frequently used by many organizations when preparing a budget for a single or multiple projects (Shtub et al., 1994, p. 381). The procedure begins by setting aside resources destined for different *project categories* in a way that reflects the organization's strategic goals. For example, the organization may decide on threshold amounts of resources in each technological area for the purpose of maintaining its capabilities in these areas. The procedure then continues down the hierarchy of the organization by making internal allocations within each category. For example, a government agency that is engaged in R&D projects will allocate budgets to divisions that specialize in different technological areas. These divisions, in turn, will allocate resources to projects within their proficiency field.

The initial allocation of resources may produce sub-optimal solutions since it is done without regard to specific knowledge on the candidate projects. Suppose that each division, in the government agency example, is given enough resources to conduct one major project. But what if the first division has three candidate projects that have greater potential benefits than the first choice opportunities of the other divisions? In this case, implementing a strict top–down scheme will lead to a sub-optimal solution.

The iterative approach tries to eliminate this shortcoming. After resources are allocated, we measure deviations between project requirements and allocated resources and modify the allocations—first at the lower end of the organizational hierarchy (divisions, in the above example) and then at higher levels (ultimately, at the top organizational level). This process may be repeated several times until an acceptable solution is reached (for an elaborate discussion of the process see Meredith and Mantel, 2003, Chapter 7).

This step is desirable in our context for at least two reasons. First, it ensures that the allocation of inputs reflect the organizational strategic considerations. Second, it decomposes the portfolio selection problem into smaller problems, with sub-groups of projects that are more homogeneous. The individual project evaluation, which is discussed in the next step, is more appropriate in such conditions.

3.2. Step 2: Individual project evaluation

After the initial budget allocation, the focus shifts to the project level. Each group of projects, to which resources were allocated at the previous step, is treated separately. The projects within each group are evaluated via the DEA–BSC model discussed earlier, where each project is represented by the inputs assigned to it and the outputs that are expected from it and additional restrictions are added to represent the desired balance requirements among the outputs. The inputs and outputs in the model are defined in a standardized way so as to enable their accumulation in a later step. For example, the input set may include man-hours (possibly broken down by skills), value of equipment and or materials to be purchased, etc. and the output set may contain expected monetary value, probabilities of technical and or market success, etc.

3.3. Step 3: Projects variability control

Portfolios are more than mere collections of projects—they possess characteristics of their own. The variability of the risks associated with the individual projects in a portfolio is one such characteristic. To control the variability of the project's characteristics, we compute indices that are associated with each project, and restrict their variability by setting thresholds. We use three indices—risk, efficiency and balance: (1) The risk index for a project is the product of the project's overall budget and the probability that the project will *not* succeed. (2) The efficiency index for a project is the efficiency score computed by the CCR model (that is, without any balance constraints). (3) The balance index for a project is computed as a ratio of the DEA–BSC score and the CCR score of that project.

We associate minimal thresholds with the efficiency and balance indices and a maximal threshold with the risk index, and use them to screen the initial list of candidate projects. The remaining subset of the candidate list contains the projects whose efficiency, risk and balance indices exceeded the threshold values. There is a delicate tradeoff in setting these thresholds. On one hand, we would like to set higher efficiency and balance-thresholds and lower risk-thresholds so as to guarantee that each of the remaining candidate projects meets our requirements. On the other hand, such thresholds reduce the number of candidates and hence the number of alternative portfolios. This may be counter-productive since a better balance can be achieved at the portfolio level by allowing different projects to compensate for each other (e.g., mixing high-risk, medium-efficiency projects with low-risk, high-efficiency projects).

3.4. Step 4: Generation of portfolios

The purpose here is to generate alternative portfolios for subsequent evaluation. We define a *complete portfolio* as a union of smaller portfolios associated with the different groups of candidate projects, according to the categories mentioned in step 1. To demonstrate the generation procedure, let $G = \{1, 2, \ldots, n\}$ denote any particular group of candidate projects, and Q_k the group of projects in portfolio k taken from $G\,(Q_k \subset G)$. Let the vector z_k represent the particular selection of projects in portfolio k ($z_{jk} = 1$, if project j is included in portfolio k; otherwise $z_{jk} = 0$).

The generation procedure focuses on input requirements and availabilities. Let x_{ij} be the amount of input i required for project j, and R_i the total availability of the same input. The amount of input i allocated to portfolio k is denoted by $\hat{x}_{ik}$; it is calculated by applying an accumulation function on the input values of the projects in the portfolio. The accumulation function is discussed in depth in the next section. For the purpose of current presentation we will assume here the following simple input-accumulation function:

$$\hat{x}_{ik} = \sum_{j=1}^{n} x_{ij} z_{jk}.$$

Definition. A portfolio k with the set $Q_k \subset G$ of projects is said to be *maximal* with respect to group G and resource availabilities $R = \cup_i R_i$, if the following two conditions hold:

1. The portfolio is feasible, i.e. $\hat{x}_{ik} \leqslant R_i$, $\forall i$.
2. Including any additional project in this portfolio would violate the resource constraints, i.e. $\forall j \notin Q_k \exists i : \hat{x}_{i\ell} > R_i$, where $Q_\ell = Q_k \cup \{j\}$.

In order to generate alternative maximal portfolios from group G, we use a branch-and-bound procedure, as illustrated in Fig. 1. We start at node 0 with an empty portfolio. Node 0 represents the problem of generating maximal portfolios from group G. The portfolio associated with this node is an empty portfolio. From node 0, we branch to n sub-problems (represented by nodes $1, \ldots, n$), by adding each time a single project to the portfolio according to the order of their indices. Node i at the first level of the branching process represents the problem of generating a maximal portfolio with project i included, and projects with lower indices, $\{j \in G, j < i\}$, excluded. The portfolio associated with this node includes only project i. This branching rule defines a partition of the general problem into disjoint sub-problems, while their unity defines the original problem. This partition rule may be further applied to the sub-problems of the original problem.

We use the following notation: above the arrow we designate the vector z_k for the corresponding portfolio, and below the arrow we designate the vector $\{\hat{x}_{ik}, i = 1, \ldots, m\}$ of inputs for the corresponding portfolio. A bound is set in one of two cases: (1) after the project with the highest index, n, is included (the

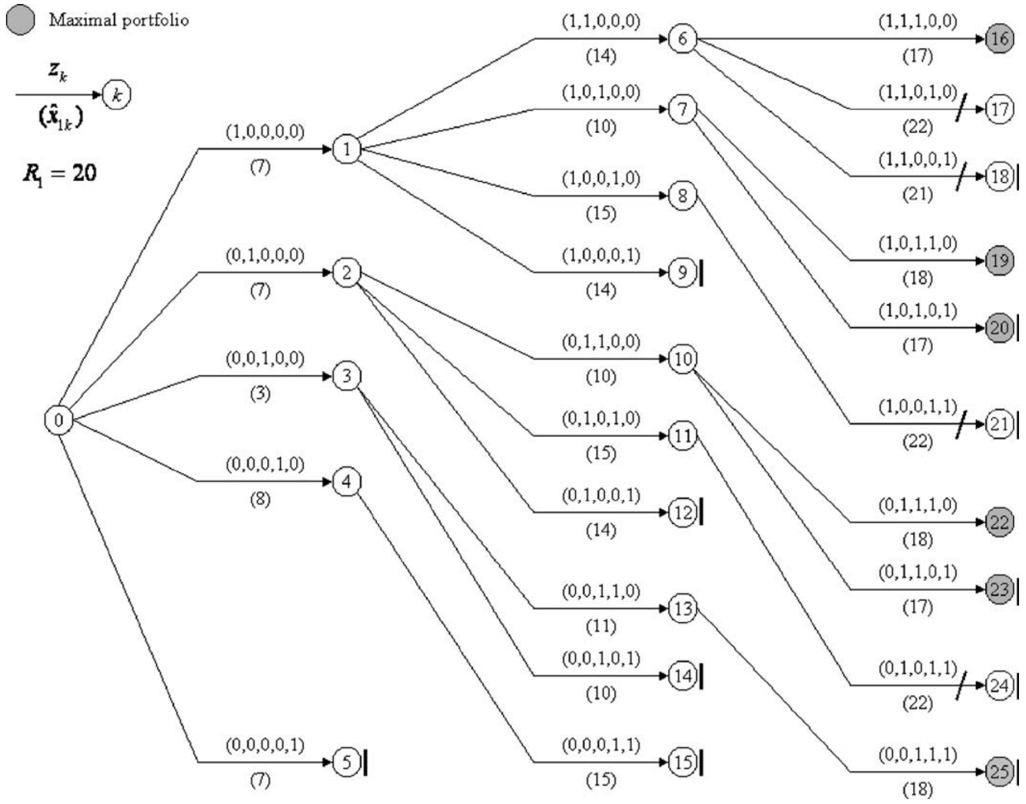

Fig. 1. An example of a branch-and-bound procedure that generates all possible portfolio alternatives ($n = 5$, $m = 1$, $R_1 = 20$).

bound in this case is designated by the vertical line), or (2) when at least one of the resource constraints is violated, meaning that $\exists i : \hat{x}_{ik} > R_i$ (designated by the diagonal line). In the first case, the bound is set after the node, meaning that the portfolio represented by the node should be further considered. In the second case, the bound is set before the node, meaning that the portfolio represented by the node is non-feasible, and should not be further considered. This procedure, when fully exploited, creates a list of all possible portfolios. A shadowed filling, in Fig. 1 designates the nodes corresponding to maximal portfolios.

3.5. Step 5: Applying an accumulation function to determine the inputs and outputs of the candidate portfolio

In this step we model the maximal portfolios as virtual DMUs with specified inputs and outputs. To compute the values of inputs and outputs for a portfolio as a whole, we use an *accumulation function* that is applied on the inputs and outputs of the individual projects included in the portfolio. The accumulation function is first presented for the simple case, where no interactions among the projects are assumed, and later for the more general case that involves such interactions.

3.5.1. An accumulation function without interactions

The amount of input i required for portfolio k, and the amount of output r expected from portfolio k for the case of independent projects without interactions, may be computed by applying a simple additive function on the individual project's inputs and outputs, as presented in Eq. (4).

$$\hat{x}_{ik} = \sum_{j=1}^{n_p} x_{ij} z_{jk}, \quad \hat{y}_{rk} = \sum_{j=1}^{n_p} p_j y_{rj} z_{jk} \quad \forall i, r, k, \tag{4}$$

where n_p represents the total number of projects.

3.5.2. An accumulation function with interactions

In practice, one can expect to find interactions among projects in a given portfolio. The subject of project interaction has long been recognized but has received relatively little attention in the R&D project literature. According to Gear and Cowie (1980) "projects may exhibit internal interactions if the resource requirements and/or benefits of one project are thought to be significantly affected in magnitude and/or timing by the selection or rejection decisions relating to one or more of the other projects in the set". On the other hand, "external interactions arise over time from overall social and economic changes which have effects that cut across many, if not all, subsets of a project set". External interaction may be treated by means of scenario analysis (i.e. identifying possible states of nature, assigning probabilities to each state, evaluating portfolios in each state of nature, and finally calculating the expected relative value of each portfolio). The types of interactions considered here are largely internal.

Internal interactions can be further classified into three categories: resource interactions, benefit interactions, and outcome interactions (Fox et al., 1984; Schmidt, 1993):

1. *Resource interactions* may occur if the total resource requirements of projects in the portfolio cannot be represented as the sum of resources of the individual projects. This is often the case when projects share resources. For example, when two related software development projects are executed simultaneously and thus share some overhead resources (e.g., personnel from the information systems department of the organization), the total amount of the overhead resources required is smaller than a situation in which the two projects are done at different times.
2. *Benefit interactions* may occur if the impacts of projects are non-additive. Typically, in this case, projects are said to be either "complementary" or "competitive". For an example of the complementary case consider an electronics company that develops a new printer/scanner and a new digital camera. Since the two are fully compatible, one can expect the total sales of these products to be larger than the sum of the individual sales had the two been developed separately. An example of competitive benefit interactions can be found in situations where product A, which partially substitutes the functions of product B developed by the same company, cannibalizes the market share of the latter, causing the combined benefit of the two products to be less than the sum of benefits of the individual products.
3. *Outcome interactions* may occur if the probability of success of a given project depends on whether another project is undertaken. The "critical mass" effect is a familiar example of this type of interaction. For example, suppose that projects A and B use the same technology. Project A requires three full time scientists, while project B requires four full time scientists. Assuming synergy among the scientists, we would expect that employing seven scientists (possibly using the same laboratory) would yield higher probabilities of success to both projects A and B as compared to allocating only three or four scientists, respectively.

To represent interactions, we generalize the model presented by Schmidt (1993) for the case of multiple inputs and outputs. This model accounts for the combined effect of resource, benefit and outcome interaction. We use it to pre-process the data for subsequent evaluation, rather than as part of a nonlinear integer program with multiple quadratic constraints, as presented by Schmidt (1993).

To account for resource interactions, let U^i be the resource interaction matrix for input i. The diagonal element u^i_{jj} represents the amount of input i required for the individual project j, that is, $u^i_{jj} = x_{ij}$. The off-diagonal element u^i_{jk} represents the resource interaction between projects j and k of input i. The resource interaction matrix is lower triangular, meaning that $u^i_{jk} = 0$, $\forall k > j$. The amount of input i required for portfolio k for the case where projects have resource interactions, is presented by the following equation:

$$\hat{x}_{ik} = z'_k U^i z_k, \quad \forall i, k. \tag{5}$$

To account for benefit and outcome interactions, let B^r be the expected value interaction matrix of output (benefit) r. The entries in the matrix B^r are constructed by multiplying the value interaction matrix of output (benefit) r, V^r, and the outcome interaction matrix P.

The entries of the value matrix V^r are defined as follows. The diagonal elements v^r_{jj} represent the output r of the individual project j, that is, $v^r_{jj} = y_{rj}$. The off-diagonal element v^r_{jk} represents the value interaction between projects j and k of output r. The value interaction matrix is also formulated as a lower triangular matrix.

The entries of the outcome matrix P describe the probability of success and the interactions between projects. The value p_{jj} is the probability that project j will succeed, $p_{jj} = p_j$, and p_{jk} is the marginal change in the probability that project j will succeed given that project k is undertaken. This definition of outcome interaction, used also by Schmidt (1993), assumes that work on project k has a positive impact on project j regardless of the outcome. In general, P is not symmetric since project j may have greater impact on project k than vice versa.

When there are no benefit or outcome interactions, the overall expected value is simply given by the sum of the expected values of the individual projects, as presented in Eq. (4). Using the new notation, we get:

$$\hat{y}_{rk} = \sum_{j=1}^{n_p} p_j y_{rj} z_{jk} = z'_k B^r z_k, \tag{6}$$

where $B^r = \overline{V^r P}$ (an over bar is used to designate a diagonal matrix). When there are no interactions, the value matrix and the outcome matrix are both diagonal.

When outcome interactions are present but there are no value interactions, then the amount of output r expected from portfolio k is given by

$$\hat{y}_{rk} = \sum_{j=1}^{n_p} y_{rj} z_{jk} \left[\sum_{i=1}^{n_p} p_{ji} z_{ik} \right] = z'_k B^r z_k. \tag{7}$$

When benefit and outcome interactions are both present, the amount of output r expected from portfolio k is given by

$$\hat{y}_{rk} = \sum_{j=1}^{n_p} z_{jk} P_j \left[y_{rj} + \sum_{i=1}^{j-1} P_i v^r_{ji} z_{ik} \right], \tag{8}$$

where P_j is the overall probability of success for project j. The probability that project j succeeds is a function of the vector z_k:

$$P_j = P_j(z_k) = \sum_{i=1}^{n_p} p_{ji} z_{ik}. \tag{9}$$

Substituting (9) into (8), we get

$$\hat{y}_{rk} = \sum_{j=1}^{n_p} z_{jk} \left(\sum_{\ell=1}^{n_p} p_{j\ell} z_{\ell k} \right) \left[y_{rj} + \sum_{i=1}^{j-1} v_{ji} \left(\sum_{\ell=1}^{n_p} p_{i\ell} z_{\ell k} \right) z_{ik} \right]. \tag{10}$$

Thus (5) and (10) provide a general accumulation function for all combined resource, benefit, and outcome interactions.

A real-world application involving internal interactions was reported by Dickinson et al. (2001). They described a portfolio management process at Boeing that involved projects to improve the product development process, to be able to eventually build a new airplane in less time and less cost, and be more competitive. A cross-functional portfolio management board (PMB) was set up with the responsibility to define the portfolio. The portfolio selection process had to account for interdependencies that were present among the projects, especially those supporting a common objective. Initially, PMB members accounted for the dependencies using qualitative assessments. Dickinson et al. (2001) then suggested the use of a dependency matrix. The elements in the matrix are values that represent the level of dependency—from values that imply that the project is entirely independent, to values that imply that the project is entirely dependent. The dependency matrix is generated jointly by the PMB members and the person (or department) that proposed the project.

3.6. Step 6: Evaluating alternative portfolios

After modeling the portfolios as virtual DMUs, we apply again the DEA–BSC model. To do so, we need to express our preferences in terms of lower and upper bounds that will enter the model. The model then yields relative values that reflect the overall attractiveness of the portfolios.

3.7. Step 7: Sensitivity analysis

Given the difficulties and limitations in assigning values to projects and portfolios, it is important to assess how sensitive is the selected portfolio to changes in value assignment. The DEA methodology provides methods to conduct such sensitivity analysis (Cooper et al., 2001). We suggest an approach presented by Beaujon et al. (2001) that relates specifically to portfolio selection. According to this approach, the standard normal random variable is used to simulate changes in values. By introducing progressively larger errors into the estimates, a less accurate evaluation process is simulated. The selected portfolio, when using the adjusted data, may now achieve lower attractiveness ratings. We may also select the best portfolio for each set of data, and compare the various portfolios on the basis of their robustness to changes in the value estimates. Other sensitivity analyses may be performed on the initial allocation of resources, on threshold settings at the screening stage, and on preference assignment through balance limit parameters. We may also conduct neighbor search, by replacing one project at a time with projects that are not included in the portfolio. The resulting portfolios may be added to the DEA analysis and estimated with reference to the established "production function" produced by the DEA.

4. Numerical example

We use the following example to illustrate some of the numerical aspects of the proposed methodology. The example is drawn from a decision-making problem in a large governmental agency charged with selecting and supporting technological projects. The data is randomly generated to avoid any confidentiality difficulties.

The particular organization discussed here has a hierarchical structure, with divisions responsible for selecting and supporting R&D projects in specific technological areas. We shall assume that the overall R&D budget of the organization was already allocated to the divisions in a way that reflects overall organizations' strategic goals (Step 1). Henceforth, we will assume having one group of projects, associated with computer technology and managed by the professional division in the organization that specializes in this technology. We shall also assume that fixed amounts of resources were allocated to the division to pursue its objectives. Notice that the set of projects we consider in the division level (as opposed to the entire set of projects considered in the organizational level) is more homogeneous, by the fact that the corresponding projects belong to the same technological field.

The computer technology division, in our example, is considering 15 R&D projects (e.g. vocal e-mail, user tracking in mobile communication, interactive learning, etc.). The data includes measures classified to the following 4 BSC-like cards: *economic contribution* (through improved quality and productivity and cost reduction), *scientific contribution* (in the sense of better use and rapid diffusion of existing scientific knowledge, and advancing the body of scientific knowledge), *social contribution* (in terms of job creation and better working conditions), and *resource requirements* (in terms of personnel, material, etc.).

These cards are generally relevant to governmental agencies and not-for-profit enterprises charged with R&D projects. Similar criteria were used in Oral et al. (1991) to evaluate R&D projects for the iron and steel industry in Turkey. To obtain estimates for these criteria they used groups of experts for each project (including managers, engineers, planners and researches), and applied a version of the Delphi method as presented, for example, by Martino (1983). In other implementations, these cards may require multiple quantitative and qualitative measures. DEA can handle cases with many inputs and outputs measured in different scales, as demonstrated by many previous publications (Cooper et al., 2000).

In order to simplify the presentation, we will henceforth assume only 1 or 2 measures in each of the cards. Specifically, we associate 1 output measure with each of the economic, scientific and social contribution cards, and 2 input measures with the resource requirements card. The inputs include: work content in full time equivalent (FTE) terms (x_{2j}), and material costs in monetary terms (x_{1j}). The outputs include: economic contribution (y_{1j}), measured in monetary term; scientific (y_{2j}) and social (y_{3j}) contributions, both measured on a scale of 0–100 (as in Oral et al., 1991). A probability of success (p_j) is also associated with each project (see Martino, 1994, pp. 185–190, for different approaches to obtain such probability estimates).

The interactions among the resources, value and outcomes are included in the resources interaction matrices, U^1 and U^2; the value interaction matrices, V^1, V^2, and V^3; and the outcome interaction matrix P. The inputs, outputs and probability of success of each project, and their interactions for the set of 15 projects in our example, are listed in Table 1. The project interactions are relevant when the portfolios are generated, and are discussed later on when we apply Step 5 of the methodology.

Using the data in Table 1, we first apply the DEA–BSC model on the individual projects (Step 2). To ensure that the attractiveness scores produced by the model reflect the desired balance of outputs, we set values for the lower and upper bounds $[L_k, U_k]$ (short notation for L_{C_k} and U_{C_k} in Eq. (3)). This is in fact a value judgment. Usually, such limits are arrived at by seeking a consensus among policy makers (top management executives) as to the relative importance of the outputs. In our example, we set these bounds to be $[0.2, 0.6]$ for all 3 output categories (or cards), meaning that the proportion of the total output (or the importance attached to each card) could be as low as 0.2 and as high as 0.6.

The resultant attractiveness ratings of the projects are presented in a decreasing order in Table 2. Projects with relatively low attractiveness ratings become candidates to be screened out. In our example projects 9 and 15 with an attractiveness rating of less than 0.3 are screened out at this step.

To generate alternative portfolios, we calculate indices for each project: efficiency-index, balance-index and risk-index (Step 3). These indices give more insights into the evaluation of the individual projects, and enable us to control some characteristics of the portfolios that are generated. Using these indexes we can explain the relatively low attractiveness rating of projects 9 and 15 that where screened out in the previous step. Project 9 has a high probability of success of 0.8 and modest resource requirement (11 FTEs and 13 K$), and therefore a low risk index. However, its contribution is relatively low in all three cards, making its efficiency rating relatively low. Project 15 has a higher risk index than project 9. However, its relative

Table 1
Data for simulated example[a]

Project	Inputs		Outputs			Probability of success
	#1[b]	#2[c]	#1[d]	#2[e]	#3[f]	
1	10	8	158	30	40	0.6000
2	11	18	3101	90	95	0.3000
3	**114**	**5**	**1240**	**70**	**20**	**0.6000**
4	13	7	137	10	20	0.7000
5	54	21	1312	90	40	0.7000
6	63	7	429	95	25	0.8000
7	49	20	785	95	20	0.9000
8	19	4	276	15	10	0.5000
9	**11**	**13**	**85**	**10**	**10**	**0.8000**
10	**111**	**3**	**1700**	**90**	**95**	**0.4000**
11	99	15	985	35	90	0.9000
12	**35**	**9**	**382**	**25**	**15**	**0.5000**
13	74	14	516	70	95	0.5000
14	**22**	**8**	**218**	**20**	**10**	**0.6000**
15	**36**	**9**	**25**	**20**	15	**0.7000**

FTE interactions	Mat. cost interactions	Econ. cont. interactions	Sci. cont. interactions	Social cont. interactions	Outcome interactions
$u^1_{2,1} = -1$	$u^2_{4,3} = -1$	$v^1_{4,1} = 370$	$v^2_{4,2} = 5$	$v^3_{4,2} = 15$	$p_{1,5} = 0.2$
$u^1_{3,2} = -4$	$u^2_{5,2} = -4$	$v^1_{6,2} = 940$	$v^2_{5,1} = 10$	$v^3_{6,5} = 40$	$p_{2,3} = 0.5$
$u^1_{4,1} = -2$	$u^2_{7,5} = -1$	$v^1_{7,3} = 200$	$v^2_{9,4} = 50$	$v^3_{9,1} = 30$	$p_{8,3} = 0.1$
$u^1_{8,3} = -1$	$u^2_{12,8} = -2$			$v^3_{14,6} = 30$	$p_{13,11} = 0.3$
$u^1_{11,7} = -15$					

[a] The data of projects that were screened prior to the construction of the alternative portfolios are designated by bold fonts.
[b] Work content (in full time equivalent terms, FTE).
[c] Material costs (in monetary terms, thousands \$).
[d] Economic contribution (in equivalent monetary terms, thousands \$).
[e] Scientific contribution (estimate in a scale from 1 to 100).
[f] Social contribution (estimate in a scale from 1 to 100).

contribution and especially the economic contribution are fairly low. It also has a low balance index and an overall lowest attractiveness rating.

To control the portfolio characteristics we may set limits to the variability of the individual project's indices. In this example, we will allow efficient score of 0.5 or higher, balance of 0.5 or higher [1], and risk less than \$200,000 (calculated by assuming \$5000 per FTE, and multiplying the total input with the probability of failure). As a result of these limits, 4 projects are screened out: projects 12 and 14 for low efficiency index, and project 3 and 10 due to their high risk index. We shall proceed from this step onward with the remaining 9 projects from which we will construct and evaluate feasible portfolios. The order of the projects in the portfolio is according to their indices, starting with the lowest at the left hand side of z_k.

As mentioned earlier, the R&D projects have overlaps that come to play when portfolios are evaluated. For example, the overlaps allow FTE savings if more than one project is selected. In our example, project 1

[1] By restricting the balance index we contribute to the robustness of the resulting portfolio in the sense that while some of the projects will not succeed, we will still maintain some balance of outputs. A balance index of 1.0 means that the project is within the balance boundaries set earlier for the single projects.

Table 2
Results for the simulated data: sorted projects[a]

Project	Attractiveness rating	Project indices		
		Efficiency	Balance	Risk
2	1.0000	1.0000	1.0000	51
6	1.0000	1.0000	1.0000	64
10	1.0000	1.0000	1.0000	**335**
11	0.8133	1.0000	0.8133	51
5	0.8101	0.8433	0.9606	87
7	0.8051	1.0000	0.8051	27
1	0.6996	1.0000	0.6996	23
3	0.6540	0.9687	0.6751	**230**
13	0.6362	0.7769	0.8189	192
4	0.5381	0.6325	0.8508	22
8	0.5102	0.5224	0.9766	50
14	0.3726	**0.3772**	0.9878	47
12	0.3606	**0.3611**	0.9986	92
9	**0.2413**	0.3393	0.7112	14
15	**0.1191**	0.3521	**0.3383**	57

[a] Project numbers and the corresponding values that exceed the thresholds are in bold.

overlaps with project 2 so 1 FTE can be saved if both projects are funded. Similarly, projects 4 and 1 have an overlap that provides a saving of 2 FTEs if both are funded. The overall FTE requirements of the projects and their interactions are conveniently presented by the following matrix.

$$U^1 = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4 & 114 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 54 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 63 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 49 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 111 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -15 & 0 & 0 & 0 & 99 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 35 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 74 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 36 \end{bmatrix}.$$

The diagonal entries of the matrix represent the FTE requirements of the individual projects (from Table 1), and the off-diagonal elements represent the interactions. Note that this matrix includes the data of all 15 projects that we started with. However, when projects are screened out, the corresponding lines and columns of the matrix should be deleted. The material costs interactions can be also represented in a similar way with matrix U^2 (whose elements are derived from Table 1). The data in matrix notation is presented in Appendix A.

The projects also have interactions among the outputs. For example, if projects 1 and 4 are both successful there will be an additional effect—an increase in profits of $370,000, and if projects 2 and 6 are both successful there will be an increase in profit of $940,000. The individual project economic contribution and their interactions are represented by matrix V^1. In a similar way matrices V^2 and V^3 present the project scientific and social contributions.

The probability of success of the individual projects and their interactions are represented by matrix P. The main diagonal entries are the individual projects probabilities of success, and the off-diagonal entries represent the outcome interactions (technical dependence). For example, projects 1 and 5 have an outcome interaction, in our example. Project 1 considered alone has a probability of success of 0.6. However, work on project 5 interacts with project 1 so that if project 5 is funded, the probability of success of project 1 will increase to 0.8. Thus, the outcome interaction of project 5 on project 1 is 0.2, as indicated by the off-diagonal entry in P.

Table 3
Results for the simulated data: sorted portfolios[a]

Sorted portfolios	z_k	$\hat{x}_{1k}$	$\hat{x}_{2k}$	$\hat{y}_{1k}$	$\hat{y}_{2k}$	$\hat{y}_{3k}$	Attractiveness rating
1	111100100	104	54	2416	128	106	1.0000
2	111110000	148	57	2847	197	144	1.0000
3	000011011	270	56	2349	249	195	1.0000
4	000110011	290	57	2561	220	227	1.0000
5	010110010	227	57	3304	191	175	1.0000
6	110110100	156	54	2682	196	131	0.9943
7	010010111	266	58	2936	198	206	0.9839
8	101110010	237	58	2578	200	197	0.9764
9	100100011	237	58	2344	173	217	0.9761
10	100001011	217	57	2101	191	199	0.9699
15	110100010	173	58	2862	144	165	0.9422
20	101011010	217	57	2282	218	157	0.9303
25	110011100	151	57	2438	214	91	0.9041
30	101110001	212	57	1949	204	164	0.8860
35	010101100	133	58	2693	176	75	0.8759
40	011100010	177	57	2831	123	150	0.8657
45	000101110	206	59	2649	181	132	0.8428
47	111010101	187	58	2241	172	138	0.8200
48	001011101	218	52	1542	211	105	0.8107
49	001110101	223	53	1754	182	137	0.8020
50	010100101	158	53	2245	126	105	0.7697
51	011100001	152	56	2203	126	117	0.7653
52	011001001	147	59	1991	156	107	0.7651
53	101100101	168	54	1744	135	127	0.7644
54	000101101	196	58	2021	184	99	0.7542
55	010001101	153	56	2033	155	95	0.7468
56	101001101	163	53	1449	153	109	0.7242

[a] The vector z_k represents a particular selection of projects from the reduced set of nine candidate projects including the following projects $(1, 2, 4, 5, 6, 7, 8, 11, 13)$. For example, the first portfolio in this table includes projects (1, 2, 4, 5, and 8).

Now, we apply the branch-and-bound procedure on the remaining group of projects under consideration (Step 4). We shall assume that the resources available to the computers division are 300 FTE and $60,000 for materials, and we will apply equations (5) and (10) to calculate the accumulated inputs and outputs, taking into account the interactions (Step 5).

The total number of maximal portfolios under these conditions is 56, with portfolios including 4 to 6 projects. Having calculated all maximal portfolios, including their accumulated inputs and outputs, we apply the DEA–BSC model again, this time to evaluate the portfolios (Step 6). Table 3 presents the sorted portfolios (top 10 ranked portfolios, lower 10, and some portfolios with intermediate ranking). For each portfolio the table gives its identification (by a 9-digit binary vector), the corresponding accumulated inputs and outputs, and the attractiveness ratings.

Eventually, we have achieved 5 portfolios with the highest attractiveness ranking of 1, and 10 portfolios with a ranking higher than 0.95 (see Table 3). In the top 5 portfolios we find two portfolios, #1 and #2, that include 5 projects each and differ in only one project; and 3 portfolios, #3, #4 and #5 that include 4 projects each and differ again in only one project.

Portfolio #1 includes projects 1, 2, 4, 5 and 8. Project 2 is a natural choice for any portfolio since it has a high expected economic contribution, relatively low resource requirements, and an overall attractiveness rating of 1 (seen in Table 2). In fact, it appears in 5 out of the 10 top ranked portfolios. In portfolio #1 it interacts with projects 1, 4 and 5, causing total resource reduction and an increase in both scientific and social contributions. Portfolio #2 includes project 6 instead of project 8 (in portfolio #1). Project 6 has a relatively high probability of success (0.8), high economic and scientific contributions and relatively low material costs, giving it (like project 2) the highest attractiveness ranking of 1. It also appears in 7 out of the 10 top ranked portfolios. In portfolio #2 project 6 also interacts with projects 2 and 4 causing high economic and social contributions. Portfolios #3 and #4 demonstrate the effect of outcome interaction. These portfolios include projects 11 and 13. The probability of success of project 13, in this case, is increased from 0.5 to 0.8. This positive outcome interaction affects the total ranking of these portfolios. On the lower end of Table 3 we find projects that have a relatively low attractiveness rating, and do not interact. Portfolio #56, for example include projects 1, 4, 7, 8 and 13 which have relatively lower attractiveness raking and do not interact.

Finally, we note that the analysis does not point out a single best portfolio. Rather, it reduces the large number of potential portfolios to a small manageable number of alternative best choices, while integrating many seemingly different criteria.

5. Summary and future research

This paper describes a methodology for R&D portfolio analysis in which effectiveness, efficiency, and balance considerations can be integrated. The methodology is based on relative evaluation of entities (projects or portfolios), and uses an evaluation model that was inspired by an integrated DEA–BSC model that was first presented by Eilat et al. (2004).

The approach in this paper may serve as an alternative to the conventional multi-dimensional knapsack approach, which applies a mixed integer-programming model to find an optimal portfolio with respect to a well-formulated objective function and multiple resource constraints. The approach here is not restricted to finding an optimal solution or to one objective, but rather to evaluate alternative portfolios in the presence of multiple objectives and possible interactions among the projects.

The methodology was designed to accommodate uncertain and subjective data. This is usually what is available in such decision problems. It also allows for comparison of alternatives without requiring strict weights or conversion factors among variables, and it can combine qualitative, intangible data, together with quantitative data.

Naturally, several possible extensions of our methodology can be studied. Throughout the paper, we focused attention on the static portfolio selection problem. One possible extension is to extend the model to a dynamic environment. This is feasible since the static problem is the foundation for the dynamic one. Dynamic changes in the portfolio can be evaluated by means of sensitivity analysis.

Another important and challenging improvement is to develop a method, and corresponding error bounds that allows using only a representative group of portfolios as a reference set in the portfolio generation process (Step 4), rather than exploring all possible portfolios from a reduced set of candidate projects. This method will assist in processing large databases of projects and evaluating a large variety of portfolios.

It might be also important to allow different levels of implementation of the same project, and process them as different project alternatives. This option will generate not only the desired portfolio, but it will also indicate the optimal level of implementation. A similar idea was introduced by Burnett et al. (1993) in the context of an application to the Gas Research Institute (GRI).

The effect of returns-to-scale of a portfolio was not discussed in this paper. Naturally, this effect may be nonlinear (introducing more resources may yield less or more than linear returns). We may produce a measure of the scale effect by generating contours of production functions, and evaluating the distances between these contours. For example, when no interactions between projects are assumed, it is easy to show that all portfolios reside in the envelope of the candidate projects. Hence, in this case we may evaluate the portfolios with respect to the project's envelope.

Appendix A: The data in matrix notation

$$U^2 = \begin{bmatrix}
8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -4 & 0 & 0 & 21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9
\end{bmatrix},$$

$$V^1 = \begin{bmatrix} 158 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1240 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 370 & 0 & 0 & 137 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1312 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 940 & 0 & 0 & 0 & 429 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 200 & 0 & 0 & 0 & 785 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 276 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 85 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1700 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 985 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 382 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 516 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 218 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 \end{bmatrix},$$

$$V^2 = \begin{bmatrix} 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 35 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix},$$

$$V^3 = \begin{bmatrix} 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 40 & 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 95 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 90 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 95 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 15 \end{bmatrix},$$

$$P = \begin{bmatrix} 0.6 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 \end{bmatrix}.$$

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