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Theory and Methodology

Optimizing the assignment of aircrews to aircraft in an airlift operation

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Abstract: The problems of constructing flight crew schedules, in the military as well as in the civilian arena, are usually so complex that efficient heuristics rather than optimal solutions are sought. For a special case of an airlift operation, this paper provides a simple exact algorithm to determine the minimal number of crews required to be stationed at the various bases where the operation takes place. The paper also indicates, and illustrates via a simple example, how this algorithm can serve as the basis for some heuristics which will address the more complex crew scheduling problems.

Keywords: Military operations research; Airlift operations; Aircrew scheduling

1. Introduction

Planning and executing a military airlift operation which involves the movement of large number of personnel and large quantities of material is a challenging task. The US Air-Force Military Airlift Command (MAC) is authorized to plan such operations and execute them when necessary. In a recent publication, [8], Solanki and Southworth describe various aspects of the airlift planning problem. They divide the input data to the problem into five categories: movement requirements, aircraft resources, airfield resources, aircrew resources and the network structure of the flight legs. The problem they formulate is too complicated for practical solution by an optimal algorithm. Hence, an insertion heuristic is used to update routes and schedules of aircraft so as to execute the movement of freight and personnel according to the movement requirements while meeting the various constraints.

This paper focuses on the issue of determining the minimal number of crews required to run a military airlift and the decision of were should they be stationed. We consider an airlift operation which consists of several *routes*, each having *missions* which are subject to given time schedules. The distinction

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between a mission and a route is that the latter is a purely physical entity, whereas a mission is a route with the associated attributes of aircraft and starting time. It is required that the scheduling period (cycle) for each mission will commence and terminate at the same location which is called 'home base' (not necessarily the same home base for different missions). It is further assumed that within the scheduling cycle, each aircraft completes exactly one mission. [1]

The aircraft are manned with aircrews that are required to rest for a certain period of time after each leg of a mission. A mission may be continued whenever a rested aircrew is available at the location. Each aircrew can be assigned any leg of any mission and can substitute any other aircrew provided it has rested as required. Given the number of missions that are needed to be flown on the different routes, and the scheduled timetable that is associated with those missions, we consider the following problems:

(1) What is the minimum number of crews that are needed to maintain the operation?
(2) How many aircrews are needed to be staged at each location?
(3) If the number of available aircrews is less than the minimum needed, which legs of what missions may be delayed so that the minimum required number of aircrews is reduced?

We will exclude from the analysis the trivial case where the rest period of the aircrews is shorter than the period of time during which the aircraft are delayed. If this is the case, then it is clear than the minimum number of aircrews is equal to the number of missions and all the aircrews must be staged initially at their respective home bases.

2. Time-expanded network

It is assumed that all changes in the system take place at integer multiples of a basic time unit, which will be taken as 1 for simplicity. Thus, the scheduling cycle is discretized so that the system is fully described by its states at times 0, 1, 2,....

Suppose the airlift operation covers L locations (bases) B^l, $l = 1, \ldots, L$, and is scheduled over a cycle of T time periods. Since the number and the schedule of all missions are given, the aircrews' scheduling problem is fully determined by the departure times of the legs of the missions and by the availability of aircrews in the various locations at the different time periods. A time-expanded network that depicts the aircrews' scheduling problem may be constructed as follows:

'Aircrew' network

The following definitions provide a description of the network which depicts the travel of crews throughout the airlift.

Nodes. A node $B^l(j)$ represents location B^l, $l = 1, \ldots, L$, at time period j, $j = 1, \ldots, T$.

Arcs. Two types of arcs are defined in this network; inter-location arcs and intra-location arcs. An *inter-location* arc has the form $(B^k(i),\ B^l(j))$, $i + r < j$, $k \neq l$, where r is the rest period for an aircrew. [2] This arc indicates the existence of at least one aircraft that is scheduled to leave B^k at time period i and arrive at B^l at time period $j - r$. Flow on this arc represents movements of crews between the bases in the relevant time periods.

Each aircrew that has flown an aircraft on this leg is available for another flight from location B^l at time period j.

An *intra-location* arc is of the form $(B^l(i),\ B^l(j))$, $i < j$, $l = 1, \ldots, L$. An arc of this type represents available aircrews that are delayed (after finishing their rest period) from time period i to time period j at location B^l.

[1] Actually, a single aircraft may be scheduled to complete more than one mission, but we shall consider it here as a single, multiple-loop route, mission.

[2] r is fixed for all legs of all missions. The model proposed in this paper can be easily extended to cases where the rest period is of variable length.

Table 1

	from		to	
	Location	Period	Location	Period
Route 1	A	1	B	3
	B	3	C	4
	C	4	B	5
	B	5	C	6
	C	6	B	7
	B	7	A	9
Route 2	A	1	D	2
	D	2	B	6
	B	6	C	7
	C	7	A	9

For each inter-location arc, $(B^k(i), B^l(j))$, we associate a number $C^{kl}(ij)$ which is equal to the number of aircraft scheduled to depart from B^k at time period i and arrive to B^l at time period $j-r$.

A similar time-expanded network is used to describe the aircraft schedule. However, the aircraft network is solely a graphical aid and is not used in any computation.

'Aircraft' network

Nodes. $B^l(j)$ as defined for the aircrew network.

Arcs. Only the inter-location arcs of the form $(B^k(i), B^l(j))$, $i+r<j$, $k \neq l$, are used in this network. Each such arc represents a leg in the airlift mission. (Note that there is a one-to-one correspondence between an 'aircrew' arc as defined above and an 'aircraft' arc which simply defines the departure and arrival times of a leg).

Example. Suppose the airlift operation consists of two routes, four locations, and nine time periods. There is one mission defined for each route. Table 1 details the two missions along with their time table. The 'aircraft' network that depicts this operation in terms of the missions' flight legs alone is given in Figure 1.

Next, we need to adjust the above network to account for crews' rest time. Suppose the rest period after each leg is one time period. Then, the adjusted 'aircrew' network defined above has the form presented in Figure 2 (note, period 10 is the first period in the next cycle).

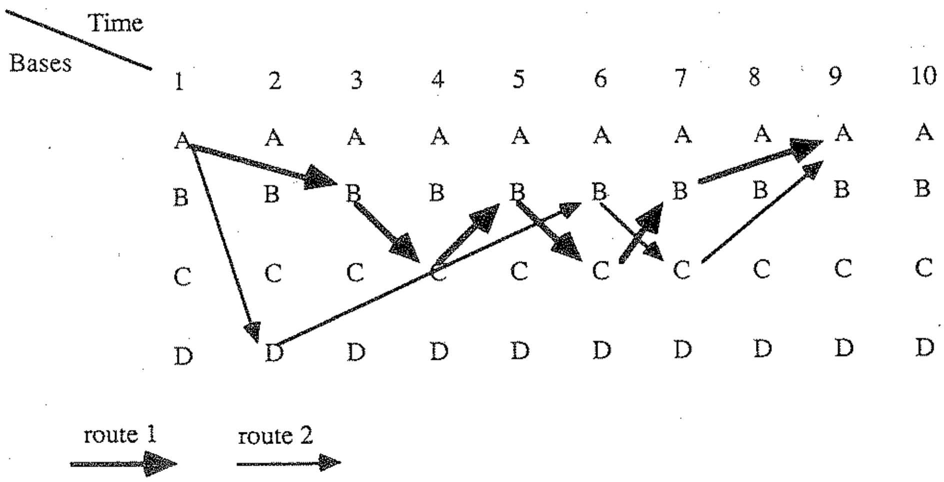

Figure 1. Aircraft network

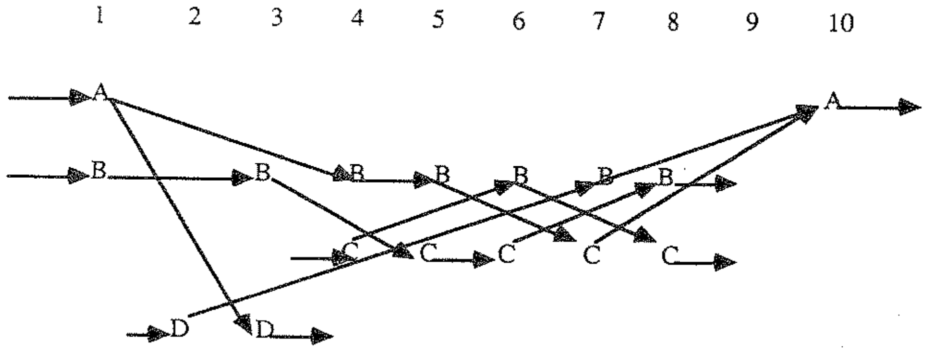

Figure 2. Aircrew network

Table 2
Aircrew assignment

	crew				
	1	2	3	4	5
from (to)	A(1), B(4) B(5), C(7) C(7), A(10)	A(1), D(3)	B(3), C(5) C(6), B(8)	C(4), B(6) B(6), C(8)	D(2), B(7) B(7), A(10)
start	A	A	B	C	D
end	A	D	B	C	A
idle	1	6	3	4	1

Table 2 illustrates a possible assignment of crews to the Aircrew network of Figure 2. For each of the five crews assigned, the number of idle periods (out of the 9 periods in the cycle) is listed in the table.

The airlift operation is cyclic in the scheduling horizons; therefore, the number of aircrews left in any location at the end of one scheduling cycle must be equal to the number needed in the same location at the beginning of the next scheduling cycle.

The problem is to find a minimum flow on the circulation network defined by the 'aircrew' network (Figure 2) with the additional arcs $(B^l(T), B^l(1))$, $l = 1, \dots, L$, as explained in Figure 3.

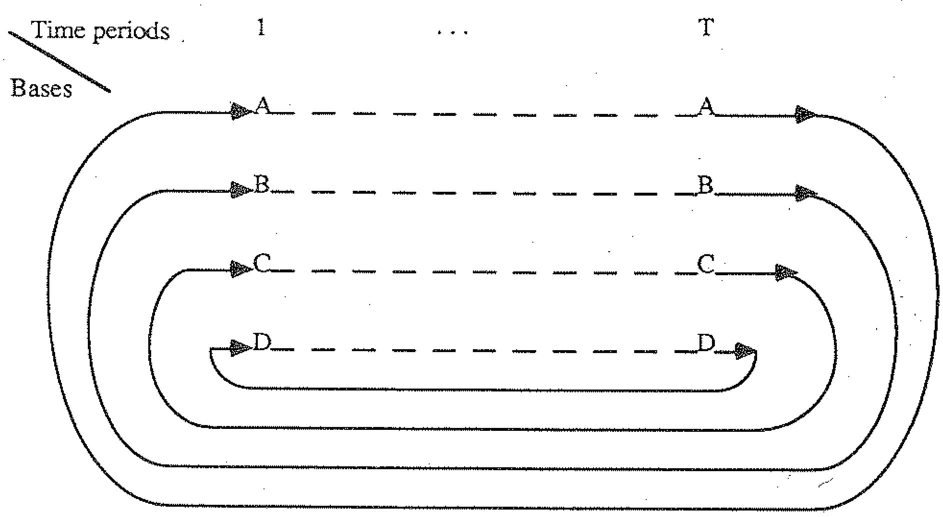

Figure 3. Minimum flow in the circulation network

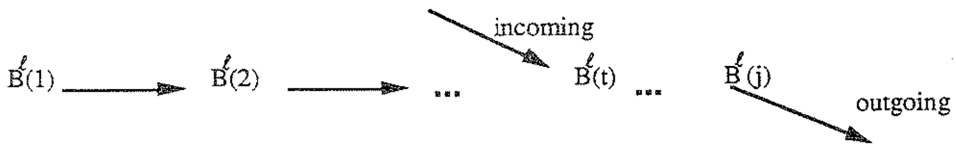

Figure 4. Incoming and outgoing arcs

The crew minimization problem may be stated by means of linear programming formulation:

$$\text{Min} \quad \sum_{l=1}^{L} f(B^l(T), B^l(1)) \tag{2.1a}$$

$$\text{s.t.} \quad \sum_{j=1}^{T} \sum_{l=1}^{L} f(B^k(i), B^l(j)) - \sum_{j=1}^{T} \sum_{l=1}^{L} f(B^l(j), B^k(i)) = 0,$$

$$i = 1,\dots,T, \quad k = 1,\dots,L, \tag{2.1b}$$

$$C^{kl}(ij) \leq f(B^k(i), B^l(j)) \leq C^{kl}(ij) \quad \forall i,j,k,l, \tag{2.1c}$$

$$f(B^l(i), B^l(j)) \geq 0 \quad \forall j > i, \quad l = 1,\dots,L, \tag{2.1d}$$

where $f(B^k(i), B^l(j))$ is the flow on arc $(B^k(i), B^l(j))$.

Problem (2.1) may be solved by an appropriate network algorithm; however, the special structure of the model can be exploited to obtain a simpler and more efficient solution procedure.

In addition, through this procedure one can identify the system's bottlenecks. This will yield insights into where and when to delay an aircraft so that the delay will reduce aircrew requirements and have a minimal effect on the airlift schedule. The details of this procedure are given in the next section.

3. A procedure for determining the minimum number of crews and their allocation

The basic principle of the following procedure is similar to the one proposed by Bartlett [1] in an algorithm for determining the minimum number of transport units to maintain a fixed schedule cyclic in time. [3] Define

$$\overline{B}^l \equiv \{B^l(1), B^l(2),\dots, B^l(T)\}, \quad l = 1,\dots,L, \tag{3.1}$$

as the set of all nodes in the adjusted 'Aircrews' network that corresponds to location B^l. We divide the set of inter-location arcs into two sets; outgoing arcs and incoming arcs. An *outgoing* arc of $\overline{B}^l$ is an arc that is rooted in one of the nodes in $\overline{B}^l$ and is connected to a node in $\overline{B}^k$, for some $k \neq l$, i.e., an arc of the form $(B^l(j), B^k(i))$, $i > j + r$. An *incoming* arc of $\overline{B}^l$ is an arc that is rooted in some node in $\overline{B}^k$, $k \neq l$, and its end point is in $\overline{B}^l$, i.e., an arc of the form $(B^k(i), B^l(t))$, $i < t - r$ (see Figure 4).

For each node $B^l(j)$ we associate a number which is equal to the excess of outgoing flow over incoming flow. In other words, the difference between the total number of flights scheduled to depart from B^l at time period j and the total number of flights scheduled to arrive to B^l at time period $j - r$. Namely,

$$\delta_j^l = \sum_{k \neq l} \sum_{i > j+r} C^{lk}(ji) - \sum_{k \neq l} \sum_{i < j-r} C^{kl}(ij) \quad \forall j,l. \tag{3.2}$$

Next, we define Δ_j^l as the *cumulative difference* between the outgoing and incoming flow at $B^l(j)$:

$$\Delta_j^l = \sum_{i=1}^{j} \delta_i^l, \tag{3.3}$$

[3] See also [5], [2], and [3] where various aspects of the original problem are discussed.

and Δ^l as the *maximum cumulative difference* at location B^l:

$$\Delta^l = \max_{j=1,\ldots,T} \{\Delta^l_j\}. \tag{3.4}$$

Property 1. *For all locations* B^l

$$\sum_{j=1}^{T} \delta^l_j = 0, \quad l = 1,\ldots,L, \tag{3.5}$$

$$\Delta^l \geq 0, \quad l = 1,\ldots,L. \tag{3.6}$$

Proof. Within a scheduling cycle all routes (missions) start and terminate at the same location (home-base), therefore the number of incoming flights in each location must be equal to the number of outgoing flights. Also, from (3.3) and (3.5) it follows that

$$\Delta^l_T = 0, \quad l = 1,\ldots,L. \tag{3.7}$$

Therefore

$$\Delta^l = \max_{j=1,\ldots,T} \{\Delta^l_j\} \geq 0. \quad \square \tag{3.8}$$

Property 2. *Δ^l is equal to the minimum number of aircrews that must be staged in location B^l at the beginning of the scheduling cycle.*

Proof. Suppose $\Delta^l = \Delta^l_{j_l} = m$; that is, the maximum cumulative excess of departures over arrivals at location l is obtained in the j_l-th time period and is equal to m. In other words, by time period j_l, m flights scheduled to leave B^l will not have crews unless m aircrews are staged at location B^l.

If we consider the m crews that are staged initially at B^l as m arrivals at time period $i = 1$, then from (3.3) and (3.4) it follows that for each time period j the difference between departures and arrivals is nonpositive, which implies that the schedule is feasible. $\square$

Property 3. *The minimum total number of aircrews needed to maintain the schedule is*

$$M = \sum_{l=1}^{L} \Delta^l. \tag{3.9}$$

Proof. Follows directly from Property 2. $\square$

As we observe in Section 2, the cyclic nature of the airlift operation problem dictates additional constraints on the system; that is, the number of aircrews left at any location at the end of a scheduling cycle should match the number needed at that location at the beginning of the next cycle. The next property shows that this requirement is satisfied by the procedure.

Property 4. *Let $B^l(t_l)$ be the last (in terms of time periods) node of $\overline{B}^l$ such that it is an end point of an inter-location arc (a node of either incoming or outgoing arcs) within a mission. For a feasible schedule,*

$$f\left(B^l(t_l),\ B^l(t_l+1)\right) = \Delta^l \tag{3.10}$$

where $t_l = T$ implies $t_l + 1 = 1$.

Proof. *Case* 1: $\Delta^l = m = 0$. No crews are stated at location B^l at the beginning of the scheduling cycle. But, from Property 1,

$$\Delta^l_{t_l} = 0. \tag{3.11}$$

Therefore $f(B^l(t_l),\ B^l(t_l+1)) = 0$.

Case 2: $\Delta^l = m > 0$. According to Property 2, at least m aircrews must be staged at B^l at the beginning of the scheduling cycle. Consider an augmented network with m arrivals at time 1. The new $\Delta_j'^l$ for the augmented network satisfies

$$\Delta_j'^l = \Delta_j^l - m, \quad j = 1, \ldots, T. \tag{3.12}$$

In particular,

$$\Delta_{t_l}'^l = \Delta_{t_l}^l - m. \tag{3.13}$$

But since $\Delta_{t_l}^l = 0$ (as a consequence of Property 1 and the fact that by definition of t_l all $\delta_j^l = 0$ for $j > t_l$), it follows that

$$\Delta_{t_l}'^l = -m, \tag{3.14}$$

which means that at the last node with either incoming or outgoing arc the cumulative difference between the flow in incoming arcs and outgoing arcs is positive $(= m)$.

To satisfy the network balance equation we must have

$$f\big(B^l(t_l),\, B^l(t_l + 1)\big) = m. \qquad \square \tag{3.15}$$

Corollary 1. *The flow on* $(B^l(T), B^l(1))$ *is equal to* Δ^l.

4. Aircraft delays

There are some tradeoffs in this model between the minimum number of aircrews needed and the delay structure of the aircraft mission.

Delaying aircraft at some locations or stretching the scheduling cycle horizon may yield a reduction in the number of aircrews. For example, consider a case with one route, two locations, flight legs of one hour between them, rest period of one hour and the mission is to fly from A to B and back in a cycle of three hours. Delaying the departure from B (while stretching the cycle to 4 hours) reduces the number of required crews from two to one. The corresponding aircrew networks are given in Figure 5.

It is, however, possible for aircraft delays to increase the number of crews required. For example, consider the airlift operation which corresponds to the aircrew network in Figure 2. The Δ^l's are:

$$\Delta^{\mathrm{A}} = 2, \quad \Delta^{\mathrm{B}} = 1, \quad \Delta^{\mathrm{C}} = 1, \quad \Delta^{\mathrm{D}} = 1 \quad \text{and} \quad M = \Delta^{\mathrm{A}} + \Delta^{\mathrm{B}} + \Delta^{\mathrm{C}} + \Delta^{\mathrm{D}} = 5.$$

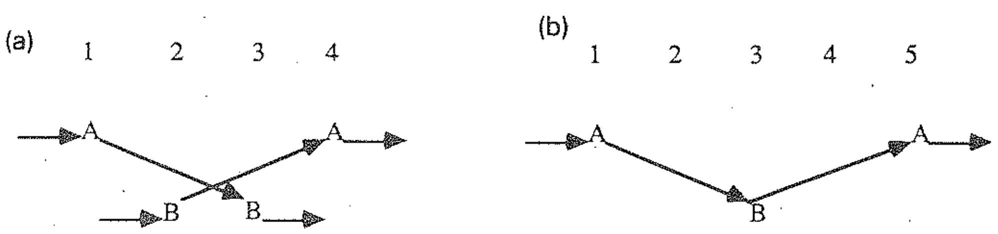

Figure 5. Aircraft delays: Reducing number of crews

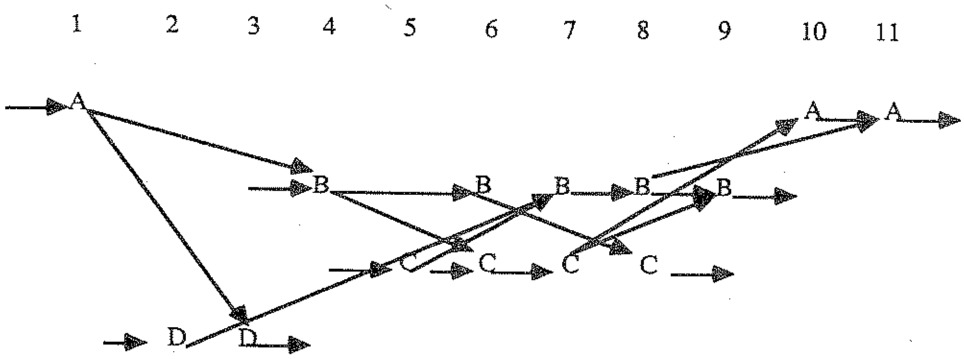

Figure 6. Aircraft delays: Increasing the number of crews

Delaying the aircraft in route 1 from B(3) to B(4), i.e., changing the mission on this route to:

from Location	Period	to Location	Period
A	1	B	3
B	4	C	5
C	5	B	6
B	6	C	7
C	7	B	8
B	8	A	10

yields the aircrew network of Figure 6.

Here, $\Delta^{\mathrm{A}} = 2$, $\Delta^{\mathrm{B}} = 2$, $\Delta^{\mathrm{C}} = 2$, $\Delta^{\mathrm{D}} = 1$ and $M = 7$.

Hence, delaying an aircraft may result in an extension of the cycle as well as an increase in the number of aircrews.

Next, it is shown that for certain locations and time periods it is impossible to reduce the number of aircrews no matter how long the aircraft are delayed. The following definitions are needed:

$$\hat{j}^l \equiv \max\left\{j:\ \Delta_j^l = \Delta^l\right\}, \tag{4.1}$$

$$\hat{j} = \max_l \hat{j}^l. \tag{4.2}$$

Property 5. *No matter how long an aircraft is delayed in $B^l(j)$, $l = 1, \ldots, L$, $j \geq \hat{j} + 1$, no reduction in the minimum number of aircrews can be achieved.*

Proof. Delaying one aircraft at location/time period $B^l(j)$ is equivalent to reducing Δ_j^l by one unit. But doing this for time periods which are past $\hat{j}l$, $l = 1, \ldots, L$, means that Δ^l is not changed for all $l = 1, \ldots, L$, and according to Property 2, no reduction in the number of aircrews needed can occur. □

5. Application to commuter planes

The procedure outlined above can be used as an approximated solution to much more complex problems which frequently arise in scheduling crews to commercial flights in the airline industry (see, e.g., [4]). These problems are characterized by a number of attributes which make the attainment of optimal solutions for them very difficult. Among these attributes we point out the following:

(1) The planning cycle is typically longer (e.g., a month).

(2) The number of airplanes, crews, 'bases', etc. is much larger.

Table 3

From	To			
	Tel-Aviv	Haifa	Jerusalem	Eilat
Tel-Aviv	–	08 10 12 16	–	08 10 16 17
Haifa	08 09 17 18	–	09 17	08 10 16
Jerusalem	–	08 18	–	10 16
Eilat	08 10 16 18	09 16 17	10 16	–

(3) The definition of a rest period is more complicated (usually, it is a function of the length of the last flight segment).
(4) Crews can be moved from one location to another, either as passengers on another flight of the same carrier (or a different carrier), or via ground transportation.
(5) There are two categories of crew: the technical crew and the flight attendants' crew. The two can be separated or kept together – dependent on circumstances.
(6) Different aircraft require different sizes of the crews (of both kinds). Also, substitution is limited as certain crews are trained only for particular types of planes.

Thus, real life scheduling of crews for commercial carriers is a difficult task. Models which attempt to describe these problems in detail contain large number of integer variables which make any exact algorithm impractical. Heuristic algorithms based on various techniques to reduce the search among feasible solutions are therefore applied.

Table 4

Time	City							
	Tel-Aviv		Haifa		Jerusalem		Eilat	
	δ_j^l	Δ_j^l	δ_j^l	Δ_j^l	δ_j^l	Δ_j^l	δ_j^l	Δ_j^l
08	2	2	2	2	1	1	1	1
09	0	2	2	4	0	1	1	2
10	0	2	−1	3	1	2	1	3
11	−1	1	0	3	−1	1	−1	2
12	0	1	−2	1	−1	0	−2	0
13	0	1	0	1	0	0	−1	−1
14	0	1	−1	0	0	0	0	−1
15	0	1	0	0	0	0	0	−1
16	2	3	1	1	1	1	3	2
17	1	4	2	3	0	1	1	3
18	−1	3	0	3	0	1	−1	2
19	−1	2	−1	2	−1	0	−2	0
20	−2	0	−2	0	0	0	0	0
Δ^l		4		4		2		3

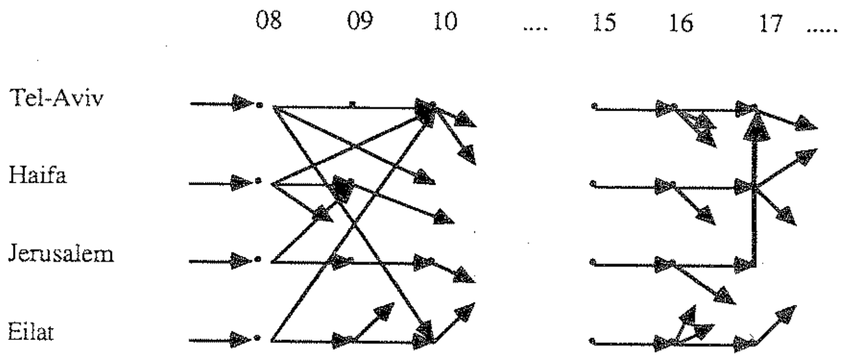

Figure 7. Illustration of vertical arcs

Some of the attributes discussed above can easily be incorporated into the procedure proposed in this paper by simple adjustments to the aircrew network structure. As an example we show above how the fourth attribute (moving crews among bases) can be handled. For illustration purposes, we use a case of a small carrier, operating domestically in Israel (the figures in the example are imaginary and do not reflect actual operations). Table 3 gives the departure times from each city (rows) to other cities (columns).

The flight duration on all segments is (rounded to) one hour, except on the Haifa-Eilat, Eilat-Haifa leg where it is (again, rounded to) two hours. Required rest period after any leg is one hour. Without allowing movements of inactive crews between cities we get the results shown in Table 4. The minimal number of crews required is 13.

Now, suppose the carrier has the option to transfer (via ground transportation) crews between Tel-Aviv and Jerusalem. Such transfer, which can be done in parallel to the rest period of the transferred crew, takes one hour. Modifying the aircrew network by allowing for inter-bases arcs starting and ending at the same time period ('vertical arcs') accommodates the possible transfer. This is depicted by Figure 7 which shows a section of the network corresponding to this example. By transferring a crew from Jerusalem to Tel-Aviv between the hours 16 and 17 (depicted in the network as a vertical arc in time 17), we are able to reduce the required number of crews (Δ_j^l) at Tel-Aviv to 3. Finally, by returning (via ground transportation) a crew from Jerusalem to Tel-Aviv at night (after the end of the schedule) we keep the balance of the number of crews in each city at the beginning of the next daily schedule.

6. Conclusions

This paper presents a simple 'accounting like' procedure which determines the minimal number of aircrews required to maintain a cyclic airlift operation. The problem is first formulated in terms of a flow network and its corresponding linear programming formulation. The new procedure is based on a single pass through the 'from-to' matrix containing the legs of the different missions. The cumulative discrepancies between the number of departing aircraft and available aircrew are recorded for each base, and the maximal discrepancy in all time periods is proven to be the minimal number of required crews needed to be staged in that base.

Finally, the problem described above is related to crew scheduling and vehicle routing problems in public transportation (see, e.g., [6,7]). The problem there is to link a number of short trips into a schedule for individual buses operating in urban areas. Typically, these problems are formulated under the constraint that each bus will begin and end its schedule in the same bus garage. When the problem is set for an inter-city schedules the additional constraint of providing the drivers a certain break intervals between successive links must be enforced so as to meet safety regulations. The time table for the buses is usually given in advance on the basis of separate models which evaluate customer's demand and the bus company capabilities. This common features may make the algorithm suggested here attractive to researchers in the public transportation area.

References

[1] Bartlett, T.E., "An algorithm for the minimum units required to maintain a fixed schedule", *Naval Research Logistics Quarterly* 4/2 (1957) 139–149.
[2] Bartlett, T.E., and Charnes, A., "Cyclic scheduling and combinatorial topology: Assignment and routing of motive power to meet scheduling and maintenance requirements. Part II: Generalization and analysis", *Naval Research Logistics Quarterly* 4/3 (1957) 207–220.
[3] Charnes, A., and Cooper, W.W., *Management Models and Industrial Applications of Linear Programming*, Wiley, New York, 1961.
[4] Gershkoff I., "Optimizing flight crew schedules", *Interfaces* 19/4 (1989) 29–43.
[5] Gleaves, V.B., "Cyclic scheduling and combinatorial topology: Assignment and routing of motive power to meet scheduling and maintenance requirements. Part I: A statement of the operations problem of the Frisco railroad", *Naval Research Logistics Quarterly* 4/3 (1957) 203–206.
[6] Hoffstadt J., "Computerized vehicle and driver scheduling for the Hamburger Hochbahn Aktiengesellschaft", in: A. Wren (ed.), *Computer Scheduling of Bus Scheduling*, North-Holland, Amsterdam, 1981, 35–52.
[7] Paixao, J., and Branco, I.M., "Bus scheduling with a fixed number of vehicles", in: J.R. Daduna and A. Wren (eds.), *Computer Aided Transit Scheduling*, Springer-Verlag, Berlin, 1988, 28–40.
[8] Solanki, R.S., and Southworth, F., "An execution planning algorithm for military airlift", *Interfaces* 21/4 (1991) 121–131.