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Decision Support

Determining all Nash equilibria in a (bi-linear) inspection game

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ABSTRACT

This paper addresses a "game" between an inspection agency and multiple inspectees that are subject to random inspections by that agency. We provide explicit (easily computable) expressions for all possible Nash equilibria and verify that none is left out. In particular, our results characterize situations when there exists a unique Nash equilibrium. We also explore special features of the Nash equilibria and the solution of the problem the inspection agency faces in a non-strategic environment.

1. Introduction

An important role that governments play in most countries is to inspect whether businesses, organizations and individuals follow the rules and regulations that are set by the legislative bodies in these countries. In most developed countries, individuals and businesses are free to run their lives or operate their businesses as they wish and accountability is based on the government's ability to inspect, whenever it wishes to do so, the compliance of any individual, business or organization with the regulations. Examples of these settings within the US include

- The Internal Revenue Service (IRS) agency selects at random some tax forms filled by individuals and investigates for possible fraud.
- The Security and Exchange Commission (SEC) checks for possible inside information incidents in trade transactions.
- The Environmental Protection Agency (EPA) inspects manufacturing plants suspected of releasing toxic gases to the atmosphere.
- The Coast Guard randomly inspects vessels entering the US territorial waters in an attempt to block drug trafficking.

Similar roles are played by international agencies such as the International Atomic Energy Agency (IAEA) that is authorized by the United Nations to inspect compliance by member states that have signed the Non Proliferation Treaty (NPT). Decisions that this agency has made in recent years concerning the location, timing and intensity of its inspections of suspected sites in various countries around the world have attracted significant public attention and have created a fair amount of controversy regarding the way these decisions were made and their effectiveness.

Early attempts to model inspection decisions using a game theoretic approach date back to work done at the Rand Corporation in the early 1960s [9]. The original model considered a two person zero-sum game between an inspector and an inspectee that has an incentive to violate some rules without being caught. If the inspectee violates the rule and is caught, s/he has to pay some penalty. In order to avoid the penalty, s/he can pay certain side-payments. Maschler [16] extended the original model by considering additional strategies for the inspectee. Thomas and Nisgav [22] extended the model to a multi-stage game and implemented it to analyze a "game" between the coast guard and marine smugglers. Diamond [8] formulated another version of a minimax model of a two-person game between an inspector and an inspectee. Baston and Bostock [7] provided a closed form solution to Thomas and Nisgav's model. This game theoretic approach was implemented in various settings including, for example, inspections for intruding forces in combat situation (see, [24]), inspections aimed at ensuring quality production in supply chains (e.g., [19,21,14]) and inspections aimed at deterring tax evasions (e.g., [18,1], the survey in [23] and references therein). Interesting psychological analysis of the logic and rationale employed by the parties in such game scenarios can be found in [17]. During the last two decades, there has been a growing public concern about the compliance of "rouge" countries with the rules and regulations of the NPT. In a sequence of papers and books that started to appear in the early 1990s, Avenhaus, Canty, Kilgour and their co-authors (see, [15,5,2,6,3,4]) as well as others (e.g., [13]) have studied game theoretical models in the context of arms-control and compliance within the NPT.

In this paper we formulate a game theory model that may be fitted to many of the real-world single inspector multiple inspectees

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scenarios discussed above. Our main contribution is in finding all Nash equilibria solutions for such games and providing explicit closed-form expressions for them. These solutions can be quite useful to inspecting agencies that wish to maximize the effectiveness of their inspections subject to the inevitable constraint of having a limited budget that can be used to execute the inspections. Our model and results extend and generalize earlier work reported in this literature. For example, [Theorem 6.4][2] studies a model that resembles our model and identifies a *single* Nash equilibrium for it (it identifies all Nash equilibria only in the degenerate case where the budget is not binding, but, without verification that none is left out). In contrast, Theorem 1 in the forthcoming Section 3 finds *all* Nash equilibria and verifies that none is left out. Also, while Chapter 9 in [2] discusses the potential for multiple equilibria, it does so only in the context of a 2-person game (that apply to the case of a single inspectee) while our analysis applies to the general multiple inspectees scenario.[1]

The rest of the paper is organized as follows. Section 2 is devoted to the introduction of the formal model. Our main contribution is given in Section 3 where we provide explicit computable expressions for all Nash equilibria (while assuring existence). Section 4 considers an inspector that operates in a non-strategic environment where the inspected parties do not select their strategies. Section 5 discusses various implications and extensions of the results of Section 3. Section 6 employs a numerical example to demonstrate the results obtained earlier. Finally, Section 7 summarizes the paper and points out directions for future research.

2. Formulation of the basic model

The basic model consists of a system which has $n+1$ players – an inspection agency I and n parties $P_1,\ldots,P_n$. The parties are subject to regulatory requirements and each party decides whether to comply with or violate the restrictions imposed on it. The inspection agency is responsible for carrying out inspections aimed at verifying that the parties comply with its regulations. Since the resources available to the inspection agency are usually limited, it cannot perform full inspections on all the parties. The decision problem of the agency is to determine which parties will be inspected and how much resources to invest in each inspection. The following table summarizes the payoffs to the inspection agency I and each party P_i as a function of their (binary) actions – a pair (u,v) in the table represents payoff u to I and payoff v to party P_i.

Let $N\equiv\{1,\ldots,n\}$. Restrictions on the quadruples (A_i,B_i,C_i,D_i) are:

$$C_i<0<D_i \quad\text{and}\quad B_i<A_i \quad \text{for } i\in N. \tag{1}$$

The restriction $C_i<0<D_i$ means that when P_i cheats, it benefits if I does not inspect, and it loses if I does inspect. The restriction $B_i<A_i$ means that if P_i cheats, the inspection agency has a higher return if it conducts an inspection than if it does not (it may still lose due to inspection costs).

We note that $A_i>B_i>0$ means that when P_i cheats, I has a positive gain, whether or not it inspects P_i (compared to 0 gain when P_i cooperates). In such cases, I has an incentive to induce cheating. There are various anomalies about this case and some of them are discussed in Sections 5 and 6. Realistic situations for an "inspection game" will have $B_i<0<A_i$ (e.g., a plant that releases toxic gas when there are no EPA inspections) or $B_i<A_i<0$ (i.e., I loses when P_i cheats, whether or not it inspects P_i, but, inspecting leads to smaller losses). Still, the results of the next section (the existence and computation of Nash equilibria) hold under the (weak) conditions of (1).

[1] Surprisingly, the results in [2] are more general and comprehensive than those reported in [4].

Table 1
The payoff table corresponding to party i.

$I\backslash P_i$	Violation	Compliance
Inspection	(A_i,C_i)	(0,0)
No inspection	(B_i,D_i)	(0,0)

The zero values in the right column of Table 1 mean that a complying party is indifferent to whether or not it is inspected (i.e., the inspection does not disrupt its operations) and the cost of the inspection to I is negligible compared with A_i and B_i. These assumptions are relaxed in Section 5.

Formally, an inspection game is an $n+1$ person game; we denote the players $0,\ldots,n$, referring to player 0 as the *inspection agency* (I) and to players $1,\ldots,n$ as *parties* (P_i) that are subject to inspection. The decision variables for I are the intensity levels of inspecting the different parties, denoted $z_1,\ldots,z_n$, respectively. Each z_i can take values in the unit interval $[0,1]$ and is interpreted as the "proportion" of full inspection that I allocates to party P_i ($z_i=1$ represents full inspection and $z_i=0$ represents no inspection). These "proportions" may be interpreted in more than one way – for example, they may represent ratios within given time intervals in which the parties will be under inspection (see [3]). In the sequel, we assume that choosing $z_i=1$ requires I to allocate W_i resources to P_i while choosing a smaller proportion (i.e., $0\leqslant z_i<1$) means an allocation of W_iz_i resources. We further assume that the inspection agency is restricted by a finite budget W. Thus, I is to select a vector $z\in Z$ where

$$Z\equiv\left\{z=(z_1,\ldots,z_n)\in\mathbb{R}^n : 0\leqslant z_i\leqslant 1 \quad\text{for } i\in N \quad\text{and}\quad \sum_{i\in N}W_iz_i\leqslant W\right\}.$$

Note that for W to be meaningful, it has to satisfy $\sum_{i=1}^{n}W_i>W$. The decision variable y_i for each party P_i, $i=1,\ldots,n$, is assumed to be within the unit interval $[0,1]$. Like the z_i's, the y_i's can also be interpreted in more than one way – for example, y_i may represent the probability that P_i will cheat. In the sequel, we view each y_i as a decision (rather than a random) variable that represents the level of cheating that P_i decides to follow ($y_i=1$ means complete violation; smaller values $0<y_i<1$ mean partial violation and $y_i=0$ means full compliance). We further assume that each party decides whether to comply with or violate the regulations independently of the other parties.[2] Thus, each party P_i is to select an element from

$$Y^i\equiv\{y\in\mathbb{R}:0\leqslant y\leqslant 1\}.$$

When each party $i\in\{1,\ldots,n\}$ selects, respectively, action $y_i\in Y^i$, we use the notation y for $(y_1,\ldots,y_n)$. Also, we let $Y\equiv Y^1\times Y^2\times\cdots\times Y^n$, be the joint set of actions for all n parties.

The utility of I depends on the action $z\in Z$ that it selects and on the joint actions $y\in Y$ selected by the P_i's – it is expressed by

$$U^0(z,y)\equiv\sum_{i=1}^{n}y_i[z_iA_i+(1-z_i)B_i]. \tag{2}$$

The utility of P_i, $i=1,\ldots,n$ depends on the action y_i that it selects and on the action z_i selected by I – it is expressed by

$$U^i(z_i,y_i)\equiv y_i[z_iC_i+(1-z_i)D_i]. \tag{3}$$

We recall that a joint set of actions for the $n+1$ players is a *Nash equilibrium* if no player can benefit from deviating from his/her strategy while the other players' strategies are held fixed. If such a set is represented by $(z^*,y^*)\in Z\times Y$, the formal conditions are:

[2] Notice, however, that Nash equlibria imposes implicit dependencies among the parties as will be demonstrated in Section 6.

$$U^0(z^*,y^*) = \max_{z\in Z} U^0(z,y^*), \quad \text{and} \tag{4}$$

$$U^i(z_i^*,y_i^*) = \max_{y_i\in Y^i} U^i(z_i^*,y_i) \quad \text{for } i=1,\dots,n. \tag{5}$$

3. Characterizing and computing the Nash equilibria

In the current section we present our main result – a closed form expression of all Nash equilibria of the inspection game defined in Section 2.

To streamline the analysis, we scale variables and data by setting

$$x_i \equiv W_i z_i,\quad a_i \equiv \frac{A_i}{W_i},\quad b_i \equiv \frac{B_i}{W_i},\quad c_i \equiv \frac{C_i}{W_i},\quad d_i \equiv \frac{D_i}{W_i}\quad \text{for } i=1,\dots,n. \tag{6}$$

With these changes, I is to select a vector $x \in X$ where

$$X \equiv \left\{ x=(x_1,\dots,x_n)\in \mathbb{R}^n : 0\leqslant x_i \leqslant W_i \text{ for } i=1,\dots,n \text{ and } \sum_{i=1}^n x_i \leqslant W \right\}$$

and the utility functions for the inspection agency and for the n parties are now expressed, respectively, by

$$U^0(x,y) \equiv \sum_{i=1}^n y_i[x_i a_i + (W_i - x_i)b_i], \quad \text{and} \tag{7}$$

$$U^i(x_i,y_i) \equiv y_i[x_i c_i + (W_i - x_i)d_i]. \tag{8}$$

Of course, Nash equilibria of the original problem are in one-to-one correspondence with those of the transformed problem; a Nash equilibrium for the latter is a pair $(x^*,y^*)\in X\times Y$ satisfying

$$U^0(x^*,y^*) = \max_{x\in X} U^0(x,y^*), \quad \text{and} \tag{9}$$

$$U^i(x_i^*,y_i^*) = \max_{y_i\in Y^i} U^i(x_i^*,y_i) \quad \text{for } i=1,\dots,n. \tag{10}$$

The data of the transformed problem consists of real numbers $\{a_i, b_i, c_i, d_i, W_i : i = 1,\dots,n\}$ and W; in particular, (1) becomes

$$d_i > 0 > c_i \quad \text{and} \quad a_i > b_i \quad \text{for } i=1,\dots,n. \tag{11}$$

Also, for simplicity of the exposition we assume henceforth that the $(a_i - b_i)$'s are distinct and that the parties are indexed so that

$$a_1 - b_1 > a_2 - b_2 > \cdots > a_n - b_n > 0. \tag{12}$$

Our main result provides a characterization of all Nash equilibria of the transformed game.

Theorem 1. *The following conditions are necessary and sufficient for $(x^*,y^*)\in X\times Y$ to be a Nash equilibrium:*

(a) *If $\sum_{i=1}^n \frac{W_i d_i}{d_i - c_i} < W$ then*

$$\frac{W_i d_i}{d_i - c_i} \leq x_i^* \leq W_i \quad \text{and} \quad y_i^* = 0 \quad \text{for } i=1,\dots,n; \tag{13}$$

in particular, in this case multiple Nash equilibria exist.

(b) *If $\sum_{i=1}^{k-1} \frac{W_i d_i}{d_i - c_i} < W < \sum_{i=1}^{k} \frac{W_i d_i}{d_i - c_i}$ for $k\in\{1,\dots,n\}$ then*

$$(x_i^*,y_i^*) = \begin{cases} \left(\frac{W_i d_i}{d_i-c_i}, \frac{a_k-b_k}{a_i-b_i}\right), & \text{if } i=1,\dots,k-1,\\ \left(W - \sum_{i=1}^{k-1}\frac{W_i d_i}{d_i-c_i}, 1\right), & \text{if } i=k,\\ (0,1), & \text{if } k<i\leqslant n; \end{cases} \tag{14}$$

in particular, in this case there is a unique Nash equilibrium.

(c) *If $W = \sum_{i=1}^{k} \frac{W_i d_i}{d_i - c_i}$ for $k\in\{1,\dots,n\}$ then*

$$(x_i^*,y_i^*) = \begin{cases} \left(\frac{W_i d_i}{d_i-c_i}, \frac{u}{a_i-b_i}\right), & \text{if } i=1,\dots,k,\\ (0,1), & \text{if } k<i\leqslant n, \end{cases} \tag{15}$$

for some $a_{k+1} - b_{k+1} \leqslant u \leqslant a_k - b_k$, with $a_{n+1} - b_{n+1} = 0$; in particular, in this case multiple Nash equilibria exist.

The next lemma records necessary conditions for Nash equilibria to be used in the proof of Theorem 1.

Lemma 1. *Suppose $(x^*,y^*)\in X\times Y$ is a Nash equilibrium. Then*

(a) $\left[x_i^* < \frac{W_i d_i}{d_i-c_i}\right] \Rightarrow [y_i^* = 1]$,

(b) $\left[x_i^* > \frac{W_i d_i}{d_i-c_i}\right] \Rightarrow [y_i^* = 0]$,

(c) $\left[x_j^* < \frac{W_j d_j}{d_j-c_j}\right] \Rightarrow [x_i^* = 0 \text{ for every } j<i\leqslant n]$,

(d) $\left[x_j^* < \frac{W_j d_j}{d_j-c_j} \text{ for some } j\right] \Rightarrow \left[x_i^* \leqslant \frac{W_i d_i}{d_i-c_i} \text{ for every } i\right]$, *and*

(e) $\left[\sum_{i=1}^n \frac{W_i d_i}{d_i-c_i} \geqslant W\right] \Rightarrow \left[\sum_{i=1}^n x_i^* = W\right]$.

Proof

(a): Suppose $x_i^* < \frac{W_i d_i}{d_i-c_i}$. Then $x_i^*(c_i - d_i) + W_i d_i > 0$, implying that $U_i^*(x_i^*,y_i) = y_i[x_i^*(c_i-d_i) + W_i d_i]$ is uniquely maximized over $0\leqslant y_i \leqslant 1$ at $y_i^* = 1$.

(b): Suppose $x_i^* > \frac{W_i d_i}{d_i-c_i}$. Then $x_i^*(c_i - d_i) + W_i d_i < 0$, implying that $U_i^*(x_i^*,y_i) = y_i[x_i^*(c_i-d_i) + W_i d_i]$ is uniquely maximized over $0\leqslant y_i \leqslant 1$ at $y_i^* = 0$.

(c): Suppose $x_j^* < \frac{W_j d_j}{d_j-c_j}$ and $x_k^* > 0$ for $j, k\in\{1,\dots,n\}$ with $k > j$. It then follows from part (a) that $y_j^* = 1$. An increase of x_j^* by a small amount $\Delta > 0$ accompanied with a reduction of x_k^* by Δ is then feasible and will result in a change of $U^0(x^*,y^*)$ by $[1\Delta(a_j - b_j) - y_k^*\Delta(a_k - b_k)] \geqslant \Delta[(a_j-b_j) - (a_k-b_k)] > 0$, contradicting (9).

(d): Suppose $x_j^* < \frac{W_j d_j}{d_j-c_j}$ and $x_k^* > \frac{W_k d_k}{d_k-c_k}$ for $j, k\in\{1,\dots,n\}$. It then follows from parts (a) and (b) that $y_j^* = 1$ and $y_k^* = 0$. An increase of x_j^* by a small amount $\Delta > 0$ accompanied with a reduction of x_k^* by Δ is then feasible and will result in a change of $U^0(x^*,y^*)$ by $[1\Delta(a_j - b_j) - 0\Delta(a_k - b_k)] > 0$, contradicting (9).

(e): If $\sum_{i=1}^n x_i^* < W \leqslant \sum_{i=1}^n \frac{W_i d_i}{d_i-c_i}$, then $x_j^* < \frac{W_j d_j}{d_j-c_j}$ for some j; by (a) it then follows that $y_j^* = 1$. An increase of x_j^* by a small amount $\Delta > 0$ is then feasible and will result in a change of $U^0(x^*,y^*)$ by $[1\Delta(a_j - b_j)] > 0$, contradicting (9). □

We shall use primes to refer to the negation of the implications of Lemma 1; e.g., (b') asserts that $[y_i^* > 0] \Rightarrow \left[x_i^* \leqslant \frac{W_i d_i}{d_i-c_i}\right]$.

Proof of Theorem 1

(a): Assume that $\sum_{i=1}^n \frac{W_i d_i}{d_i-c_i} < W$. To prove the "necessity" part of (13), let (x^*,y^*) be a Nash equilibrium. Assume that $y_j^* > 0$ for some $j\in\{1,\dots,n\}$ and we will show a contradiction. By (b') of Lemma 1, $x_j^* \leqslant \frac{W_j d_j}{d_j-c_j}$. We consider two cases. If $x_k^* > \frac{W_k d_k}{d_k-c_k}$ for some $k\neq j$, then (b) of Lemma 1 assures that $y_k^* = 0$; an increase of x_j^* by a small amount $\Delta > 0$ accompanied with a reduction of x_k^* by Δ is then feasible and will result an increase of $U^0(x^*,y^*)$ by $[y_j^*\Delta(a_j - b_j)] - 0\Delta(a_k - b_k)] > 0$, contradicting (9). Alternatively, if $x_i^* \leqslant \frac{W_i d_i}{d_i-c_i}$ for each $i\neq j$, then $\sum_{i=1}^n x_i^* \leqslant \sum_{i=1}^n \frac{W_i d_i}{d_i-c_i} < W$ and an increase of x_j^* by a small amount $\Delta > 0$ is feasible and will result an increase of $U^0(x^*,y^*)$ by $[y_j^*\Delta(a_j-b_j)]>0$, again, contradicting (9). So, $y_i^*=0$ for each i and (a') of Lemma 1 assures that $x_i^*\geqslant\frac{W_id_i}{d_i-c_i}$ for each i. According to I's set of strategies, $x_i^*\leqslant W_i$ for each i. This completes the verification of (13).

To prove the "sufficiency" part assume that (13) is satisfied. We then have that $U^0(x,y^*)=0$ for each x and (9) is trivially satisfied. Also,

$$U^i(x_i^*,y_i)=y_i\left[x_i^*(c_i-d_i)+W_id_i\right].$$

As (13) implies that $x_i^*(c_i-d_i)+W_id_i\leqslant 0$, we have that $U^i(x_i^*,y_i)$ is maximized over $0\leqslant y_i\leqslant 1$ at $y_i^*=0$, establishing (10). As (x^*,y^*) satisfies (9) and (10), it is a Nash equilibrium.

(b): Assume that $\sum_{i=1}^{k-1}\frac{W_id_i}{d_i-c_i}<W<\sum_{i=1}^{k}\frac{W_id_i}{d_i-c_i}$ for some $k\in\{1,\ldots,n\}$. To prove the necessity of (14), let (x^*,y^*) be a Nash equilibrium. As $\sum_{i=1}^{n}\frac{W_id_i}{d_i-c_i}\geqslant\sum_{i=1}^{k}\frac{W_id_i}{d_i-c_i}>W$, (e) of Lemma 1 implies that $\sum_{i=1}^n x_i^*=W$. So, $\sum_{i=1}^{k}\frac{W_id_i}{d_i-c_i}>W\geqslant\sum_{i=1}^n x_i^*\geqslant\sum_{i=1}^k x_i^*$, implying that $x_j^*<\frac{W_jd_j}{d_j-c_j}$ for some $1\leqslant j\leqslant k$. It now follows from (a) and (d) of Lemma 1 that $y_j^*=1$ and $x_i^*\leqslant\frac{W_id_i}{d_i-c_i}$ for each $i=1,\ldots,n$. By selecting j as the minimal index with $x_j^*<\frac{W_jd_j}{d_j-c_j}$, we will assume that

$$x_i^*=\frac{W_id_i}{d_i-c_i}\quad\text{for } i=1,\ldots,j-1. \tag{16}$$

We next observe that (c) of Lemma 1 and $x_j^*<\frac{W_jd_j}{d_j-c_j}$ imply that $x_i^*=0$ for each $j<i\leqslant n$; these equalities combine with (16) and $x_j^*<\frac{W_jd_j}{d_j-c_j}$ to show that

$$\sum_{i=1}^{j}\frac{W_id_i}{d_i-c_i}=\sum_{i=1}^{j-1}x_i^*+\frac{W_jd_j}{d_j-c_j}>\sum_{i=1}^{j}x_i^*=\sum_{i=1}^{n}x_i^*=W >\sum_{i=1}^{k-1}\frac{W_id_i}{d_i-c_i}. \tag{17}$$

It follows that $j>k-1$; as we selected $j\leqslant k$, we conclude that $j=k$ and that the asserted expressions of the x_i^*'s in (14) follow. Further, (a) of Lemma 1 implies that $y_i^*=1$ for $k\leqslant i\leqslant n$. As $U^0(x,y^*)=\sum_{i=1}^n x_i[y_i^*(a_i-b_i)]+\sum_{i=1}^n y_i^*W_ib_i$ is maximized over X at x^* and $x_i^*>0$ for $i\in\{1,\ldots,k\}$, a standard result (about the linear knapsack problem) implies that $y_i^*(a_i-b_i)$ is constant for $i\in\{1,\ldots,k\}$. Consequently, for $i=1,\ldots,k-1$, $y_i^*=\frac{y_k^*(a_k-b_k)}{a_i-b_i}=\frac{a_k-b_k}{a_i-b_i}$. This completes the proof of the necessity of (14). To prove the sufficiency part assume that (14) is satisfied. Then $y_i^*(a_i-b_i)=a_k-b_k$ for $1\leqslant i\leqslant k$ and

$$U^0(x,y^*)=\sum_{i=1}^n y_i^*[x_i(a_i-b_i)+W_ib_i] =(a_k-b_k)\sum_{i=1}^k x_i+\sum_{i=k+1}^n x_i(a_i-b_i)+\sum_{i=1}^n y_i^*W_ib_i.$$

As (12) implies $a_k-b_k>a_i-b_i$ for $i>k$, $U^0(x,y^*)$ is maximized over $x\in X$ by x^* (or any other vector in X satisfying $x_i=0$ for $i>k$). Also, for every i

$$U^i(x_i^*,y_i)=y_i\left[x_i^*(c_i-d_i)+W_id_i\right].$$

Now, for $1\leqslant i\leqslant k-1$ the bracketed term is 0, implying that $U^i(x_i^*,y_i)$ is maximized over Y^i by $y_i=y_i^*$ (or any other vector in Y^i). As $x_k^*<\frac{W_kd_k}{d_k-c_k}$, the bracketed term for $i=k$ is positive and for $k<i\leqslant n$ the bracketed term is $W_id_i>0$. Consequently $U^i(x_i^*,y_i)$ for $k\leqslant i\leqslant n$ is maximized over Y^i by $y_i=1$. We verified that (x^*,y^*) satisfies (9) and (10), consequently, it is a Nash equilibrium.

(c): Assume that $\sum_{i=1}^{k}\frac{W_id_i}{d_i-c_i}=W$ for some $k\in\{1,\ldots,n\}$. To prove the necessity of (15), let (x^*,y^*) be a Nash equilibrium. As in the proof of (b), $\sum_{i=1}^{n}\frac{W_id_i}{d_i-c_i}\geqslant\sum_{i=1}^{k}\frac{W_id_i}{d_i-c_i}=W$ combines with (e) of Lemma 1 to show that $\sum_{i=1}^n x_i^*=W$. The arguments of the proof of (b) further show that the existence of $j\in\{1,\ldots,k\}$ with $x_j^*<\frac{W_jd_j}{d_j-c_j}$ implies the modification of (17) where the last inequality "$W>\sum_{i=1}^{k-1}\frac{W_id_i}{d_i-c_i}$" is replaced with "$W=\sum_{i=1}^{k}\frac{W_id_i}{d_i-c_i}$", leading to a contradiction. Consequently, $x_i^*\geqslant\frac{W_id_i}{d_i-c_i}$ for $i=1,\ldots,k$. As $\sum_{i=1}^{k}\frac{W_id_i}{d_i-c_i}=W=\sum_{i=1}^n x_i^*$, we conclude from the nonnegativity of the x_i^*'s that $x_i^*=\frac{W_id_i}{d_i-c_i}$ for $i\in\{1,\ldots,k\}$ and $x_i^*=0$ for $i\in\{k+1,\ldots,n\}$. Next, (a) of Lemma 1 implies that $y_i^*=1$ for $i\in\{k+1,\ldots,n\}$. As $U^0(x,y^*)=\sum_{i=1}^n x_i\left[y_i^*(a_i-b_i)\right]+\sum_{i=1}^n y_i^*W_ib_i$ is maximized over X at x^* and $x_i^*>0=x_{k+1}^*$ for $i\in\{1,\ldots,k\}$ a standard result (about the linear knapsack problem) implies that $y_i^*(a_i-b_i)$ is constant for $i\in\{1,\ldots,k\}$, say u, and if $k<n$, then $u\geqslant y_{k+1}^*(a_{k+1}-b_{k+1})=a_{k+1}-b_{k+1}$. Consequently, for $i\in\{1,\ldots,k\}, y_i^*=\frac{u}{a_i-b_i}$ and $y_k^*\leqslant 1$ assures that $u\leqslant a_k-b_k$. This completes the proof of the necessity of (15).

To prove the sufficiency part assume that (15) is satisfied with a corresponding u. Then $y_i^*(a_i-b_i)=u$ for $1\leqslant i\leqslant k$ and

$$U^0(x,y^*)=\sum_{i=1}^n y_i^*[x_i(a_i-b_i)+W_ib_i] =u\sum_{i=1}^k x_i+\sum_{i=k+1}^n x_i(a_i-b_i)+\sum_{i=1}^n y_i^*W_ib_i.$$

As (12) implies $a_k-b_k\geqslant u\geqslant a_i-b_i$ for every $i>k$, $U^0(x,y^*)$ is maximized over $x\in X$ by x^*. Also, for every i,

$$U^i(x_i^*,y_i)=y_i[x_i^*(c_i-d_i)+W_id_i].$$

In particular, for $1\leqslant i\leqslant k$ the bracketed term is 0, implying that $U^i(x_i^*,y_i)$ is maximized over Y^i by $y_i=y_i^*$. Also, for $k<i\leqslant n$ the bracketed term is $d_i>0$. Consequently $U^i(x_i^*,y_i)$ for $k<i\leqslant n$ is maximized over Y^i by $y_i=1$. We verified that (x^*,y^*) satisfies (9) and (10), consequently, it is a Nash equilibrium. □

Remarks about Theorem 1:

(1) **Theorem 1 vs. Theorem 6.4 in [2]:** The closed-form solution obtained in Theorem 1 resembles the one stated by Theorem 6.4 in [2]. However, the latter identifies all Nash equlibria solutions only in the (degenerate) case when the amount of the inspection resource is not binding; and it does not verify that there are no further Nash equilibria beyond those that were identified. In all other cases, Theorem 6.4 in [2] finds just a *single* Nash equilibrium solution. Theorem 1 determines *all* Nash equilibria and verifies that none are left out. Both Theorem 1 here and Theorem 6.4 in [2] establish the existence of a Nash equilibrium.

(2) *Structure of the set of Nash equilibria and uniqueness:* Theorem 1 parameterizes the set of Nash equilibria by the amount of the inspection resource. It asserts that I sets "target-levels" $\frac{W_id_i}{d_i-c_i}$ for the inspection of P_i. Now, if the amount of the inspection resource exceeds the level that is necessary to meet all "target-levels", (i.e., $\sum_{i=1}^{n}\frac{W_id_i}{d_i-c_i}<W$), then I's equilibrium allocation is to allocate to each P_i more than its "target-level" and all parties will *fully* comply. If the overall amount of the inspection resource is "binding", i.e., $W\leqslant\sum_{i=1}^{n}\frac{W_id_i}{d_i-c_i}$, I ranks the parties by their (a_i-b_i)-values and allocates, in order

of priority, a quota $\frac{W_i d_i}{d_i - c_i}$ to their inspection; this is done until the inspection resource is depleted (the last party being allocated only a "partial quota"). A party i to which the "full quota" $\frac{W_i d_i}{d_i - c_i}$ of the inspection resource is allocated, will partially comply. The remaining parties (all but at most one will not be inspected at all) will select full violation.

Theorem 1 demonstrates, in particular, that I has a unique equilibrium strategy, except for the case where "target-levels" to each party, i.e., $\sum_{i=1}^n \frac{W_i d_i}{d_i - c_i} < W$. In the exceptional case, I allocates to each P_i any amount that exceeds its "target-levels" and "full compliance" is the only equilibrium strategy for the parties. When I's budget is limited, i.e., $\sum_{i=1}^n \frac{W_i d_i}{d_i - c_i} \geqslant W$, the equilibrium strategies of the parties are unique except for the case where the available inspection resource is any one of the "singular values" $\left\{\sum_{i=1}^k \frac{W_i d_i}{d_i - c_i} : k = 1, \ldots, n\right\}$. In these "singular" cases, the equilibrium strategies of P_i that is allocated its full "target-level" range within a specified interval and its equilibrium utility is fixed and invariant of their selection. However, in this case, the equilibrium utility of I depends on the level of compliance by the parties – the more they comply (within the permissible interval) the better off is the inspector.

(3) *Computation and Complexity:* For a given value of the budget W, the closed form solution of the set of Nash equilibria of Theorem 1 is easily computable through $O(n)$ arithmetic operation. Preprocessing of the data to get (12) will require $O(n\log(n))$ comparisons.

4. Inspector allocations in non-strategic environments

Consider the problem that I faces in *a non-strategic environment* where the inspected parties do not select their actions with consideration to actions selected by I. In these cases, the "compliance probabilities" are replaced by "no-fault probabilities" indicating the likelihood that an inspection of a certain site (party) will find that it meets safety regulations and there is no threat in it. Such situations occur, for example, in inspections concerning potential leaks from containers that store dangerous materials or cracks in gas pipes that connect vital installations.

Assume that p_i is a fixed probability of "no-fault" associated with party i. In this case, the allocation problem that I faces is the search for an optimal solution of the following LP: $\max_{x \in X} \sum_{i \in N} p_i [x_i a_i + (W_i - x_i) b_i]$ – a classic linear allocation problem. The standard solution for this problem is to rank the parties so that $p_1(a_1 - b_1) \geqslant p_2(a_2 - b_2) \geqslant \cdots \geqslant p_n(a_n - b_n)$ and allocate, in order, the minimum between the budget that is left and W_i to each party i, starting from the first party till the total budget W is exhausted.

The structure of the solution of the inspector's problem in a non-strategic environment is different in two ways from the structure of its equilibria allocation in Theorem 1. First, the priority of the firms that get a positive allocation in the conclusion of Theorem 1 is based on decreasing ranking of the $(a_i - b_i)$'s and not on the $p_i(a_i - b_i)$'s. Second, in the conclusion of Theorem 1 I sets target allocation-levels to the parties that are to be inspected which are below full capacity – the target allocation to party i being $\frac{W_i d_i}{d_i - c_i} < W_i$. Theorem 1 demonstrates that allocations at these levels suffice to induce compliance. In the non-strategic environment, I sets target allocation levels that equal the capacities, namely, W_i to party i.

The above distinction is consistent with findings reported in [11] which considered the problem of allocating a single resource for defending sites against terrorists' attacks. In that model, the sites were ranked and the allocation was based on a uniform threshold level of risk. In a non-strategic environment, the ranking was different and sites that were protected had their risk reduced to 0.

Table 1
Data for Example 1.

i	a_i	b_i	c_i	d_i	W_i	$\frac{W_i d_i}{d_i - c_i}$
1	−4	−8	−4	4	3	1.5
2	−3	−6	−3	3	4	2
3	−3	−5	−2	2	5	2.5
4	−6	−7	−6	8	7	4

5. Extensions and discussion

5.1. Non-zero returns when parties comply

We consider an extension of the model introduced and analyzed in the previous sections by replacing the $(0,0)$ entries of Table 1, respectively, with $(E_i, 0)$ (top, right) and $(0, F_i)$ (bottom, right). We note that Nash equilibria is invariant to translation of payoffs/costs to a player, hence, without loss of generality one can assume that when party i complies and is inspected, its payoff is 0 (that is, all other payoffs are stated in comparison to the payoff in that event). But, the assumption that the payoff to I when P_i complies and is not inspected is 0, independently of the party, is restrictive. Further, we will impose the assumption that for each i, $E_i < 0$ and $C_i < 0 < D_i - F_i$.

Under the more general payoffs, we now have that (2) and (3) become, respectively,

$$U^0(z, y) = \sum_{i=1}^n y_i [z_i A_i + (1 - z_i) B_i] + \sum_{i=1}^n (1 - y_i) z_i E_i$$
$$= \sum_{i=1}^n z_i [y_i (A_i - B_i) + (1 - y_i) E_i] + \sum_{i=1}^n y_i B_i \tag{18}$$

and

$$U^i(z_i, y_i) = y_i [z_i C_i + (1 - z_i) D_i] + (1 - y_i)(1 - z_i) F_i$$
$$= y_i [z_i C_i + (1 - z_i)(D_i - F_i)] + (1 - z_i) F_i. \tag{19}$$

Augmenting the change of variable and scaling implemented at the beginning of Section 3 with $e_i = \frac{E_i}{W_i}$ and $f_i = \frac{F_i}{W_i}$, (7) and (8) become

$$U^0(x, y) = \sum_{i=1}^n x_i [y_i (a_i - b_i) + (1 - y_i) e_i] + \sum_{i=1}^n y_i b_i \tag{20}$$

and

$$U^i(x_i, y_i) = y_i [x_i c_i + (1 - x_i)(d_i - f_i)] + (1 - x_i) f_i. \tag{21}$$

We note that the term $\sum_{i=1}^n y_i b_i$ in (20) and the term $(1 - x_i) f_i$ in (21) have no affect on Nash equilibria.

Incorporating $F_i \neq 0$ (without introducing the E_i's) is simple – the results of Lemma 1 and Theorem 1 remain unchanged with $d_i - f_i$ replacing d_i. In terms of the original data this means that $D_i - F_i$ takes the role of D_i; the required inequalities $C_i < 0 < D_i$ then become $C_i < 0 < D_i - F_i$, and these are satisfied by our assumption about the F_i's.

The introduction of (negative) E_i's (when nonzero F_i's are present) is more complex. In this case Lemma 1 (with $d_i \to d_i - f_i$) remains valid. As for Theorem 1, the first row of (14) becomes $y_i^* = \frac{a_k - b_k - e_i}{a_i - b_i - e_i}$; this is the case due to the requirement $y_i^*(a_i - b_i) + (1 - y_i^*) e_i = y_k^*(a_k - b_k) + (1 - y_k^*) e_k = a_k - b_k$ (the last equality following from $y_k^* = 1$). Correspondingly, the restriction on u in (15), becomes $a_{k+1} - b_{k+1} - e_i \leqslant u \leqslant a_k - b_k - e_i$.

5.2. Dependence of Nash equilibrium utility of I on W

Theorem1 allows one to determine the equilibrium utility of the inspection agency I as a function of the amount W of the inspection resource that is available. We consider the model with data that reflects the scaling introduced at the beginning of Section 3. When $\sum_{i=1}^{k-1} \frac{W_i d_i}{d_i - c_i} < W < \sum_{i=1}^{k} \frac{W_i d_i}{d_i - c_i}$, the commitment of a small amount δ of the inspection resource will be targeted to party k. It will not change the selected action of any party and it will result in an increase of $\delta(a_k - b_k) > 0$ in U^0. Thus, the marginal benefit of committing additional resource is piecewise-constant and increasing in each of the open intervals $(\sum_{i=1}^{k-1} \frac{W_i d_i}{d_i - c_i}, \sum_{i=1}^{k} \frac{W_i d_i}{d_i - c_i})$. But, when $W = \sum_{i=1}^{k} \frac{W_i d_i}{d_i - c_i}$, the actions of party P_i for $i = 1, \ldots, k$ take a range of options while the strategy of I is fixed. It follows that U^0 can take a range of values representing a "jump". When all the a_i's and b_i's are negative, this jump will be upward whereas when all a_i's and b_i's are positive, it will be downward. Thus, the equilibrium utility of I is expressed as a point-to-set function of the amount of resource W which has k jumps. This point-to-set mapping is single-valued, linear and increasing between jumps with decreasing slope in progressing intervals. But, the behavior at jumps can have increases or decreases. In particular, when the a_i's and b_i's are positive, I is better off having no resource than having it. When $W = 0$, I will not inspect anyone, all parties will cheat and I will gain $b_i > 0$ from each party P_i. When $W > \sum_{i=1}^{n} \frac{W_i d_i}{d_i - c_i}$, I's assignment of the resource to each party will exceed its "quota", all parties will comply and U^0 will be 0. This is a counter-intuitive conclusion that suggests that our model is relevant only for cases where all the A_i's and B_i's are negative.

5.3. Positive A_i's and B_i's: sub-optimality of Nash equilibrium

The fact that Nash equilibrium can be sub-optimal is a well-known phenomenon (e.g., the classical "prisoner's dilemma"). We next illustrate that under the settings assumed in Section 2, this phenomenon may be present in a very fundamental way. Assume that both A_i and B_i are positive and that the inspection resource (W) is large enough to ensure that we are under case (a) of Theorem 1. In this case, the theorem implies that I would assign the full quota to inspect each P_i and all parties would fully comply – leading to a Nash equilibrium solution of (0,0). But, since both A_i and B_i are positive, I is better off when each P_i decides to violate. Now, party P_i will violate if it knows that I will not inspect. Hence, if I does not inspect, we would get the solution (B_i, D_i) which is superior to (0,0) for both I and P_i. But, this solution is not a Nash equilibrium as I is better off inspecting P_i when P_i selects violation. This example is, in essence, a variant of the "Prisoner's Dilemma".

5.4. The role of information

In order for the parties to determine the Nash equilibria solutions (provided in Theorem 1) the data must be common knowledge. This fact has some surprising aspects that we next discuss. Theorem 1 shows that when party P_i gets a "full inspection quota" $\frac{W_i d_i}{d_i - c_i}$, its own equilibrium behavior depends on the amount W of the inspection resource that the inspection agency possesses. Specifically, if W is sufficiently large, then P_i will fully comply, that is, it will choose $y_i^* = 0$ (part (a) of Theorem 1); but, if W is limited, then P_i will only partially comply, that is, it will choose $0 < y_i^* \leqslant 1$ (parts (b) and (c) of Theorem 1). So, the equilibrium behavior of P_i depends on the amount of the inspection resource that I has. In particular, without the knowledge of W, P_i cannot determine its equilibrium behavior (even if it has access to all other data – the A_i's, B_i's, C_i's, D_i's and W_i's). This is surprising in view of the fact that the utility of party P_i depends only on its compliance level (y_i) and the amount of the inspection resource that I allocates to inspect it (x_i).

5.5. Inspector-leadership

Leadership in inspection games is a strategic concept by which, through persuasive announcement of his/her strategy, the inspector can achieve deterrence (forcing the inspectees to adhere to his/her rules and regulations), see [3]. Such leadership should not be expected under the circumstances described in this paper. In most real-world cases, the inspection resource (W) is not large

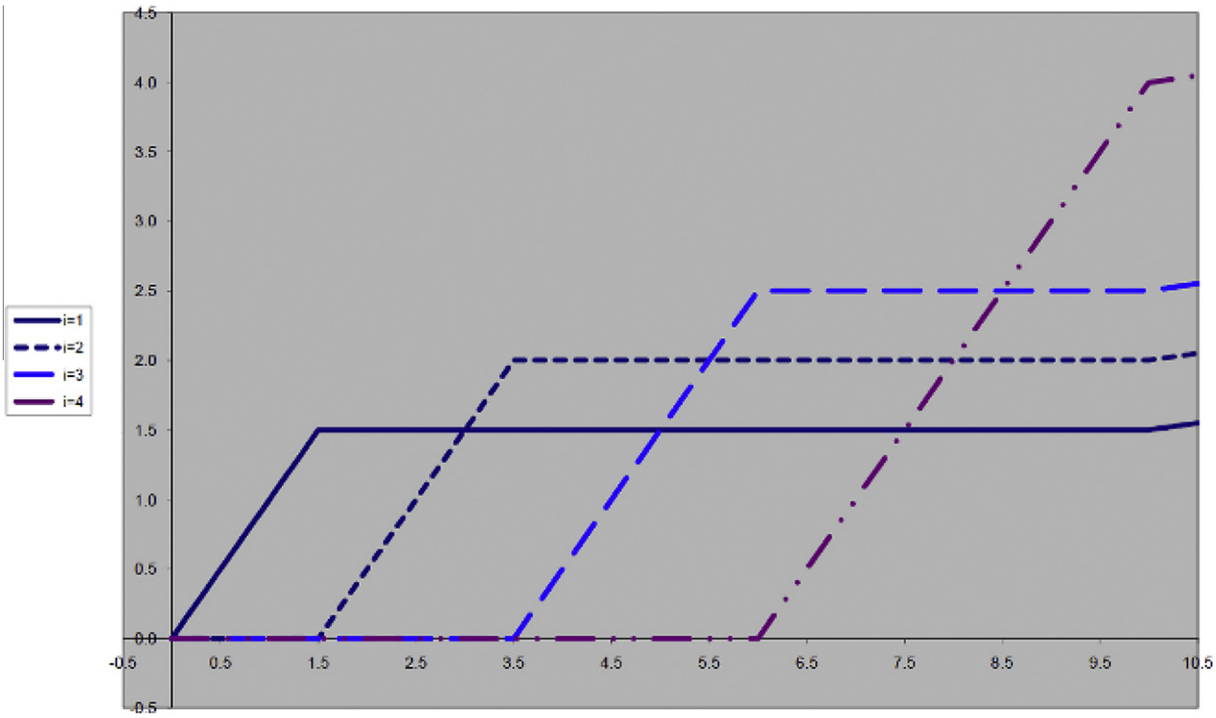

Fig. 1. Equilibrium values of x_i as a function of W.

enough to enable the allocation of the "full quota" to each P_i (that is, case (b) of Theorem 1 holds). In such circumstances, announcing its policy in advance, will only hurt *I*'s interests as it will ensure that none of the parties will fully comply with its regulations and some of them will not comply at all! This fact may explain why agencies such as the IAEA always try keep their true intentions (the sites they are going to inspect) secret until the last minute.

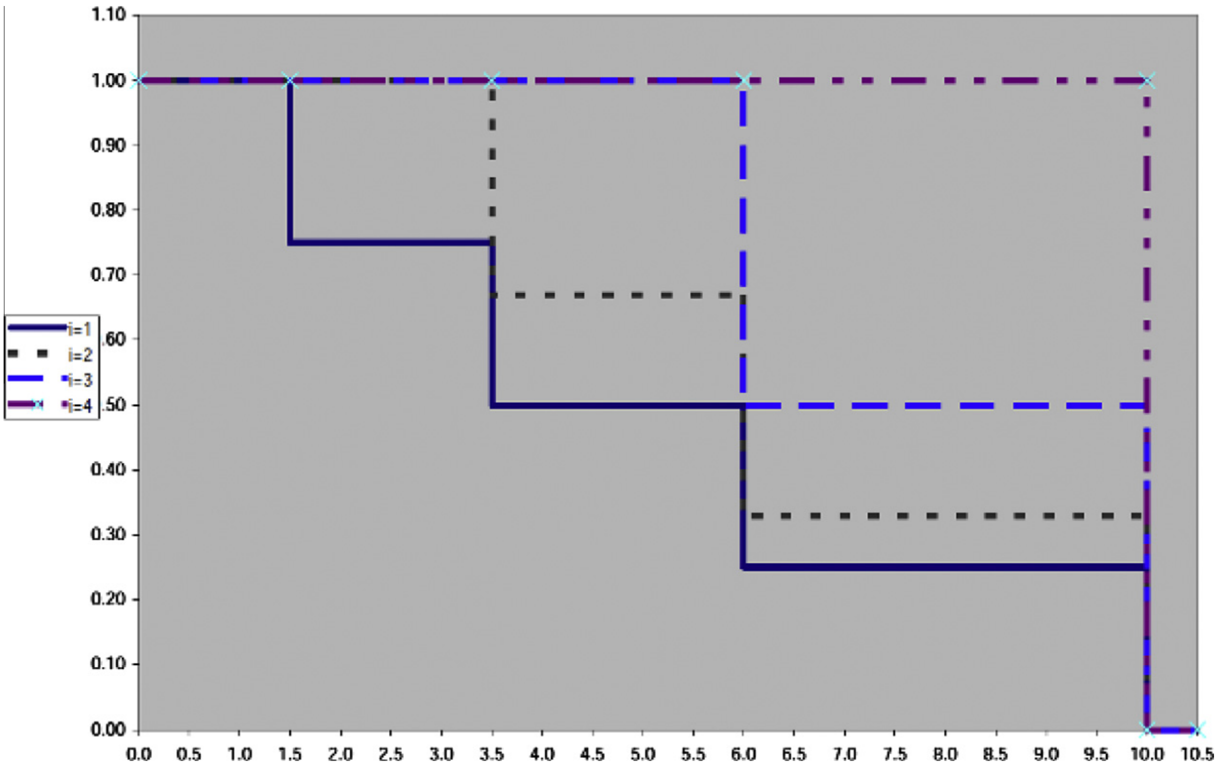

Fig. 2. Equilibrium values of y_i as a function of *W*.

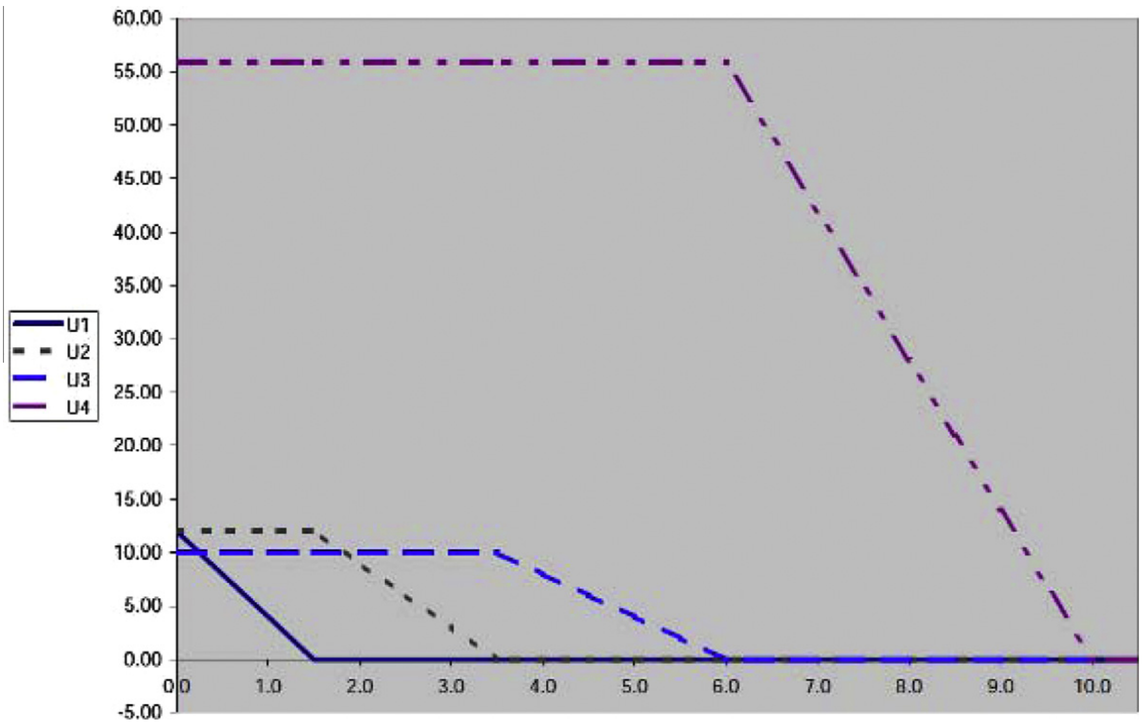

Fig. 3. Equilibrium payoffs to the inspectees as a function of *W*.

6. Numerical examples

Example 1. Consider an example with $n = 4$ and the following (scaled) data: For this example $\sum_{i=1}^{4} \frac{W_i d_i}{d_i - c_i} = 10$. Figs. 1 and 2 express, respectively, the values of x_i^*'s and y_i^*'s (the optimal strategies of the inspection agency and of the parties). The Nash equilibrium solution of this problem is depicted in Figs. 1–4, parametrically in the budget W of the inspector. We next interpret these figures partially (for $W < 3.5$). When W is small, the inspector invests its budget in the first inspectee and continues to do so until it reaches $W = 1.5$ ($x_1 = 1.5$ is the "target allocation" for inspectee 1). Under these allocations, no inspectee will comply at all, i.e., $y_i = 1$ for each i. At $x_1 = 1.5$, the first inspectee switches to a mode of partial compliance, with $0.75 (= \frac{6-3}{8-4}) \leqslant y_1 \leqslant 1$. When $1.5 < W < 3.5$, the extra resource $W - 1.5$ of the inspector are allocated to the second inspectee, the first inspectee complies partially with $y_1 = 0.75$ and all other inspectees do not comply at all, i.e., $y_i = 1$ for $i = 2, 3, 4$. When $W = 3.5$ the second inspectee switches to a mode of partial compliance, with $\frac{2}{3} \leqslant y_2 \leqslant 1$ while the first inspectee increases his/

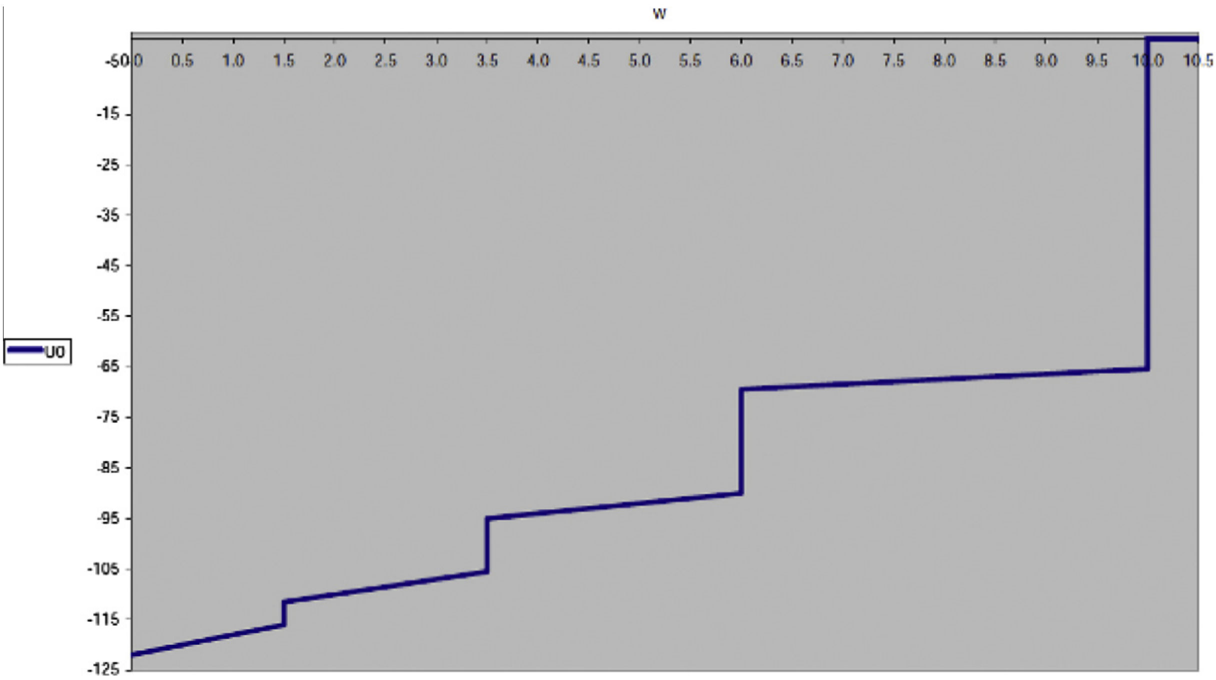

Fig. 4. Equilibrium payoffs to the inspector as a function of W.

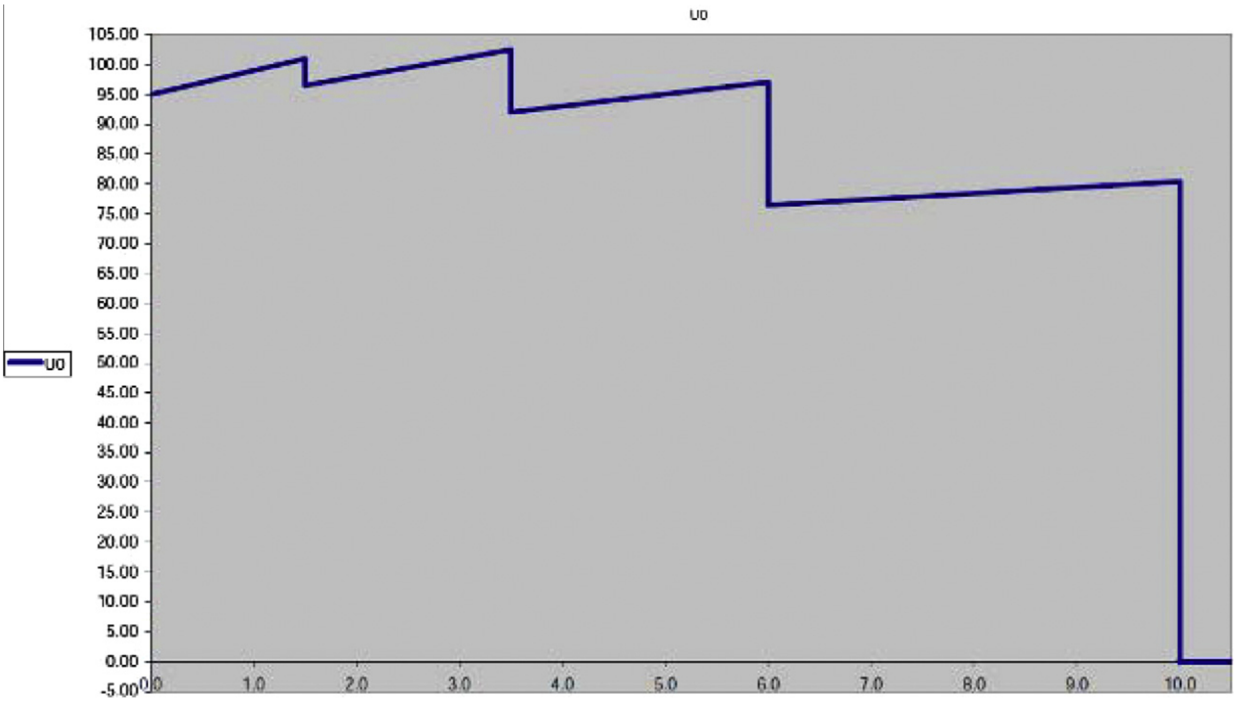

Fig. 5. Equilibrium payoffs to I as a function of W when $0 > a_i > b_i$.

her compliance to $0.5 \leqslant y_1 \leqslant 0.75$. When $3.5 < W < 6$, the extra resource $W - 3.5$ of the inspector are allocated to the third inspectee, inspectees 1 and 2 will comply partially with $y_1 = 0.5$ and $y_2 = \frac{2}{3}$, respectively, and inspecteees 3 and 4 will not comply at all. The increased compliance of the first inspectee requires some explanation. When W reaches 3.5 the inspector's return jumps (the size of the jump depends on the compliance level of the first two inspectees). When W continues to increase beyond 3.5, the inspector's rate of return is reduced. Consequently, there is an increased threat to the first inspectee that the inspector will allocate the newly available resource to inspect it, which causes the first inspectee to increase its compliance. This situation corresponds to the remark in the footnote in Section 2 about "dependence" that is induced in the equilibrium behavior of the parties. In order to prevent the inspector from increasing the resources dedicated to inspect the first inspectee, the inspectee increases its compliance level. Figs. 3 and 4 express $U^i(x_i^*, y_i^*)$'s and $U^0(x^*, y^*)$, all as functions of the inspection resource W. We note that Fig. 4 does not represent multiple values that $U^0(x^*, y^*)$ may take when $W > \sum_{i=1}^{4} \frac{W_i d_i}{d_i - c_i} = 10$.

Example 2. The data of this example is identical to that of Example 1 except that the columns of the a_i's and the b_i's are exchanged after each element in them is multiplied by -1. As a result of these changes the x_i^*'s, y_i^*'s and $U^i(x_i^*, y_i^*)$'s are the same as in Example 1, but, $U^0(x^*, y^*)$ is different and its dependence on W is expressed by Fig. 5 (again, without representing multiple values that exist when $W > 10$). Fig. 5 illustrates the anomaly of situations when the a_i's and b_i's are positive and the resultant instability in the equilibrium solutions – in particular, $U^0(x^*, y^*)$ is not monotone in W.

7. Concluding remarks

The subject of inspection games has been investigated quite extensively during the last five decades. Still, it continues to draw attention as the frequency of real-world cases in which it is relevant continues to grow. Throughout the 1960s and the three decades that ensued, the underlying motivation was the cold war between the US and the USSR and the desire to monitor the various arms control agreements that were signed by the two superpowers. Analytically, these settings led quite naturally to 2-person game formulations with various assumptions about the strategy sets that were feasible to each of the two parties.

Since the early 1990s, attention has shifted to other directions, most notably to the on-going efforts of the IAEA to stop the proliferation of military nuclear capability around the globe. Clearly, these new circumstances require moving from 2-person to n-person formulations. But, the differences do not stop here as the challenges faced by researchers that seek to analyze the new settings remain quite formidable. Our modest contribution in this paper, is to provide closed-form expressions of Nash equilibria solutions to inspection games for which no such expressions were known until now. Still, there is plenty of room for further research – some of which is pointed out below.

- In this paper we assumed that the inspectees take their decisions independent of one another. In reality, we know that collusion among some parties can and should be expected (e.g., it is quite plausible that some of the "rough" countries suspected by the IAEA in violating the NPT rules coordinated their moves in preparation for IAEA inspections).
- This paper discussed a single stage inspection game. Most, if not all, real-world inspection games are repetitive by nature. In such repetitive games, I's decisions in each stage are typically affected by the cumulative information he/she has on the compliance history of the various parties. It is not clear at all whether Nash equilibria solutions can be found for such games.
- Adopting our general model to any of the specific inspection scenarios presented in Section 1 (involving different agencies such as the IRS, IAEA, EPA, SEC, etc.) will require specific research of the data parameters and the relationship among them.

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