

Economic lot-sizing with remanufacturing options

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Received March 1999 and accepted February 2001

We study a production planning problem with remanufacturing. We provide the problem's general formulation and assess its computational complexity under various cost structures. We prove that the problem is NP-complete for general concave-cost structures. When costs are linear, we obtain an $O(T^3)$ algorithm based on transforming the problem into the transportation problem in a special way. Finally, we suggest linearizing costs as an alternative for solving the problem in the real world.

1. Introduction

We consider a single-item production system which faces periodic deterministic demand over a finite horizon and where some (or all) of the demand in certain periods can be satisfied through remanufacturing of items that were used in previous periods. Relevant information about demand for the entire planning horizon is known in advance and backlogging is not permitted. The production, holding (for both used and new items), remanufacturing, and disposal costs are given for each period.

In each period, the system needs to decide what to do with the used items at hand. There are three options: (i) discard as spoilage; (ii) keep in a "used-item storage" for future consideration; and (iii) remanufacture the items. Each of the options may be associated with some costs. Remanufacturing may involve testing, cleaning, disassembly with replacements of some components and then reassembly, etc. Holding the used items in inventory may involve shipping in and out of storage, cost of storage space, monitoring, etc. Disposal of items may incur the costs associated with transportation, disassembly and separation of hazardous materials, etc. Disposal may also be associated with negative costs (i.e., revenues) due to salvage value of the disposed items.

There are at least two categories of items for which the analysis offered here might be relevant. The first category consists of packaging and shipping materials (such as pallets or containers) used in shipments between manufacturing facilities and their corresponding distributors. The shipments are planned in advance so the demand for pallets is known. Remanufacturing the pallets means

collecting them at the distributor's facility, fixing those that were broken along the way, perhaps cleaning them and then sending them back to the manufacturer. New (and reused) pallets can be stored at the manufacturer's facility.

The second category includes repairable items (e.g., machine tools used in manufacturing). Remanufacturing corresponds to repairing the machine tools. This category has attracted considerable research in the past three decades. A typical treatment of the problem, including a discussion of the three alternative treatments of used items as given in the previous paragraph, can be seen in Simpson (1978).

It is important to note that we do not consider cases where there are limitations on how many times one can remanufacture the same item. We also do not consider perishable items or items whose quality deteriorates over time. Therefore, there is no limitation on how long an item can be carried in one of the two inventory types that are considered here. We also need to note that if demand backlogging had been allowed, the system would have become much more complex to manage. This is because we would have to keep record of how many items produced and remanufactured in each period are used to satisfy the demand in that period and how many are to satisfy the demand in previous and subsequent periods respectively in order to know how many items may be reused again.

The problem addressed in this work is different from the ones studied in the literature in both conceptual and computational aspects. Conceptually, most of the problems addressed before dealt with the flow of materials or

goods in one direction – from the supplier (or manufacturer) to the buyer (or consumer). Here we discuss a situation in which flow goes both forward (to the customer) and backward (to the manufacturer). Such situations are now denoted as “reverse logistics” (see the extensive literature reviews in Fleischermann *et al.* (1997) and Guide *et al.* (2000) and the survey on remanufacturing practices in Guide (2000)). The closest variant we find in the literature to our model is the one formulated by Richter and Sombrutzki (2000). Richter and Sombrutzki (2000, p. 311) formulate a model similar to the one we develop here and state that “*there are probably no simple algorithms to solve the general model*”. Then, they turn to analyze a special case where there is a large quantity of low-cost used products for which they are able to continue using the famous zero-inventory property that serves as the basis for all Wagner–Whitin type algorithms. They conclude by saying that “*A lot of research can be done to expand the scope of these models and algorithms*”. In this paper we take a first stab at the challenge posed in Richter and Sombrutzki (2000) and the research agenda compiled in Guide (2000, p. 478) by developing and analyzing algorithms for a general model of aggregate production planning that considers returned products.

Computationally, most of the previous research on deterministic periodic demand models assumed uncapacitated-flow. Research on capacitated-flow networks was limited to the case of production capacity. The only polynomially-solvable economic lot-sizing problem with the capacity constraints is the problem with constant capacities (Van Hoesel and Wagelmans, 1996). Problems with arbitrary production capacities were shown to be NP-hard by Florian *et al.* (1980) and also Bitran and Yanasse (1982).

Approximate solution procedures were proposed for the general capacitated problems by Bitran and Matsuo (1986) and Gavish and Johnson (1990), but their results could not be generalized. The capacity constraint in our case corresponds to the remanufacturing activity which is bounded from above (by definition, we cannot remanufacture unless items were first produced and used). Since demand is fixed, we cannot increase the cumulative remanufactured quantities beyond the cumulative demand for each period. Consequently, known results for the traditional economic lot sizing problem cannot be readily implemented in our case.

We organize the rest of the paper as follows: Section 2 gives a formal definition for the problem and provides an example which illustrates the problem. Section 3 proves the NP-hardness of the problem for general concave-costs. Section 4 presents a polynomial algorithm for the problem when costs are linear. Section 5 provides pseudo-polynomial algorithms for the problem when costs are convex and arbitrary, respectively. Finally, Section 6 points out the way to employ our results in practice and possible future research directions.

2. Problem definition and model formulation

We assume that the time horizon to be considered spans T periods. In each period t , B_t , number of used items become newly available for remanufacturing and D_t number of new items are demanded. It should be noted that although the process of returned items is usually affected by various uncertainties, firms apply forecasting techniques along with advanced information systems and even use modified MRP-like management techniques to obtain deterministic estimates for the returned quantities as is discussed by Guide *et al.* (2000). The problem is to optimally utilize the used items and plan production to satisfy the demands for the entire time horizon. With careful selection of the correlation between B_t and D_t , our formulation can model very realistic situations. For example, to model the remanufacturing of a product with a fixed k -period utilization cycle and a fixed non-reusability fraction of ρ , we simply need to set $B_t = (1 - \rho)D_{t-k}$ for every t and only let the D_t 's be input parameters for the problem.

We introduce the following definitions:

- B_t = number of used items newly available in period t ;
- D_t = number of new items demanded in period t ;
- x_t = number of newly produced items in period t ;
- y_t = inventory of new items held at the end of period t (y_0, y_T are externally given);
- z_t = number of used items being remanufactured in period t ;
- u_t = inventory of used items at the end of period t (u_0, u_T are externally given);
- v_t = number of disposed items in period t ;
- $P_t(x_t) \geq 0$: production cost in period t ;
- $H_t(y_t) \geq 0$: new item holding cost in period t ;
- $R_t(z_t) \geq 0$: remanufacturing cost in period t ;
- $W_t(u_t) \geq 0$: used item holding cost in period t ;
- $S_t(v_t) =$ disposal cost in period t .

For simplicity, we always maintain

$$P_t(0) = H_t(0) = R_t(0) = W_t(0) = S_t(0) = 0.$$

The only constraints on the variables are related to material conservations. We have the following formulation for the Production and Remanufacturing Planning (PRP) problem of minimizing the total cost of meeting the demand:

$$(PRP) \min \sum_{t=1}^T P_t(x_t) + \sum_{t=1}^{T-1} H_t(y_t) + \sum_{t=1}^T R_t(z_t) + \sum_{t=1}^{T-1} W_t(u_t) + \sum_{t=1}^T S_t(v_t),$$

subject to

$$\begin{aligned} x_t + y_{t-1} - y_t + z_t &= D_t \quad \forall t = 1, \dots, T, \\ z_t + u_t - u_{t-1} + v_t &= B_t \quad \forall t = 1, \dots, T, \\ x_t, y_t, z_t, u_t, v_t &\geq 0 \quad \forall t = 1, \dots, T. \end{aligned}$$

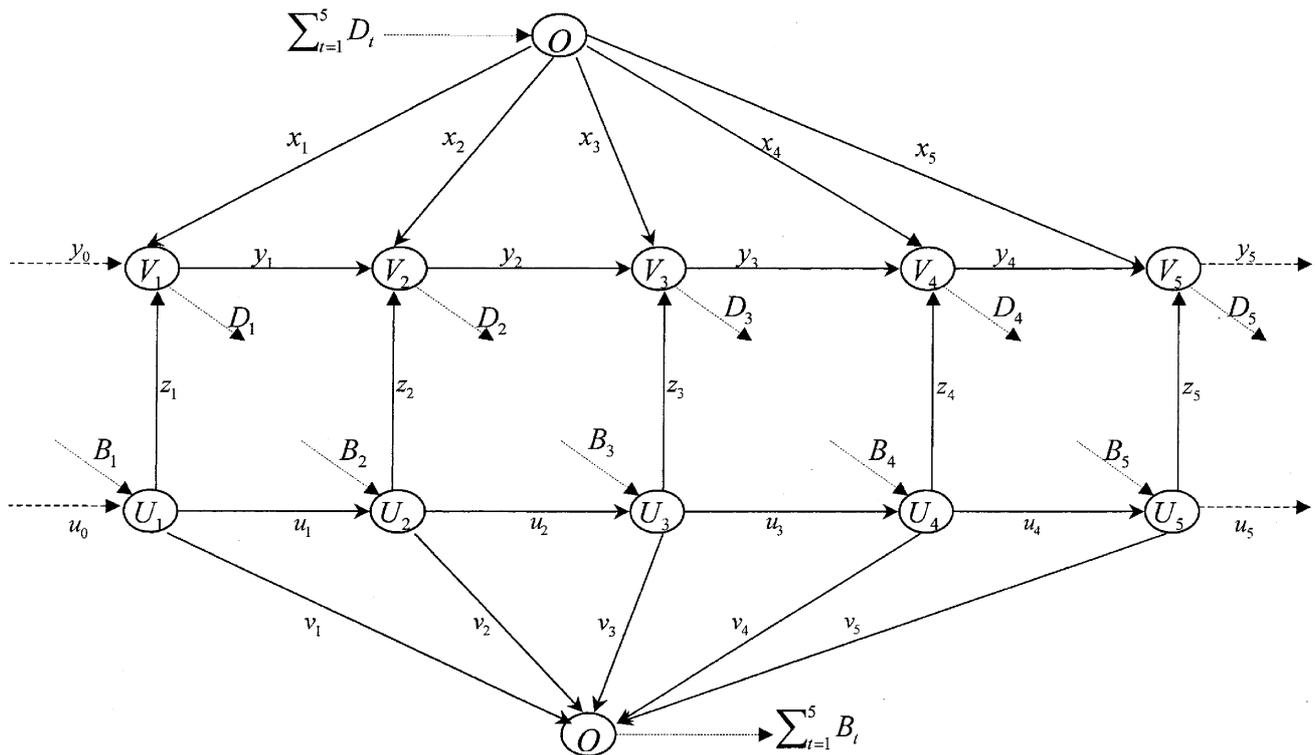


Fig. 1. Model representation.

The mathematical formulation just given is of the network flow type, as represented in Fig. 1. In the network, we have three kinds of nodes: O, U_1, \dots, U_T , and V_1, \dots, V_T . We label the arc from node A to node B as (A, B) . Hence, x_t is the flow on arc (O, V_t) , y_t is the flow on arc (V_t, V_{t+1}) , z_t is the flow on arc (U_t, V_t) , v_t is the flow on arc (U_t, O) , and u_t is the flow on arc (U_t, U_{t+1}) .

The total cost of having a certain flow in the network is the sum of costs on individual arcs. The cost function $c_{(A,B)}(f_{(A,B)})$ for each individual arc (A, B) of the amount of flow $f_{(A,B)}$ on it corresponds to one of the cost functions $P_t(f_{(A,B)})$'s, $H_t(f_{(A,B)})$'s, $R_t(f_{(A,B)})$'s, $W_t(f_{(A,B)})$'s, and $S_t(f_{(A,B)})$'s, and depends on what (A, B) represents: for example, $c_{(O,V_t)}(f_{(O,V_t)}) = P_t(f_{(O,V_t)})$. When we say a node has supply d , we mean that the difference between its total outgoing flow and total incoming flow is forced to be d . If we take node O 's supply to be $\Delta DB_{1T} = \sum_{\tau=1}^T (D_\tau - B_\tau)$, and for any $t = 1, \dots, T$, node U_t 's supply to be B_t and node V_t 's supply to be $-D_t$, then the PRP is just the minimum-cost network flow problem satisfying all the node supplies.

The following example illustrates the model and its possible outcomes.

Example

Consider five periods with known demands. Item usage time is two periods ($k = 2$) and every item becomes re-

usable (i.e., $\rho = 0$). All cost functions are assumed to have linear stationary forms as given below.

Production: $P(x) = P \times x$.

Holding (new items): $H(y) = H \times y$.

Recycling: $R(z) = R \times z$.

Holding (used items): $W(u) = W \times u$.

Disposing: $S(v) = 0$.

Assuming that production is twice as expensive as remanufacturing and that holding new items in inventory is three times as expensive as holding used items, we assign the following values to the cost parameters: $P = 10$, $H = 3$, $R = 5$, $W = 1$. Then, we solve the model for five demand scenarios where the overall demand remains fixed and the differences are caused by different demand patterns.

Scenario 1. This scenario was constructed to demonstrate cases characterized by large demands at both ends of the planning horizon and low demand in between ($D_t = \{100, 70, 50, 80, 100\}$). The scenario is given in Fig. 2 where the demands are attached to the dashed arcs emanating from the first row of nodes (that correspond to the five periods in this example). The optimal solution is shown by the values attached to the solid arcs and the objective function value is 2220.

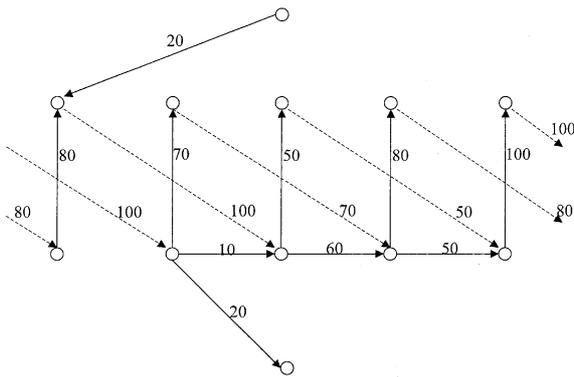


Fig. 2. Optimal solution for Scenario 1.

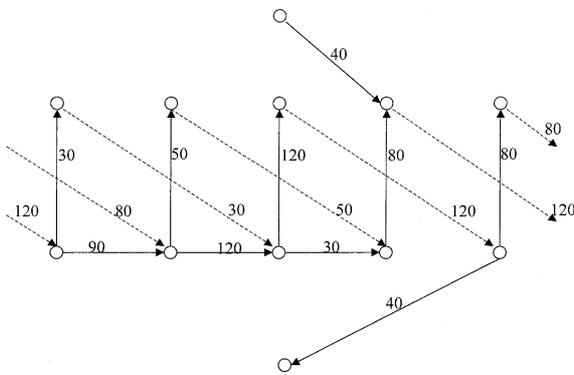


Fig. 3. Optimal solution for Scenario 2.

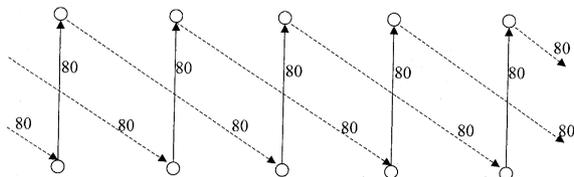


Fig. 4. Optimal solution for Scenario 3.

Scenario 2. Here we observe an opposite situation from the one depicted in Scenario 1. Low demands at both ends of the planning horizon, large demand in between ($D_t = \{30, 50, 120, 120, 80\}$). The optimal solution is shown in Fig. 3 and the objective function value is 2440.

Scenario 3. Uniform demand throughout the planning horizon ($D_t = 80, t = 1, \dots, 5$). The optimal solution is shown in Fig. 4 and the objective function value is 2000.

Scenario 4. Monotonically increasing demands throughout the planning horizon ($D_t = \{80, 90, 100, 110, 120\}$). The optimal solution is shown in Fig. 5 and the objective function value is 2650.

Scenario 5. Monotonically decreasing demands throughout the planning horizon ($D_t = \{120, 110, 100, 90, 80\}$). The optimal solution is shown in Fig. 6 and the objective function value is 3100.

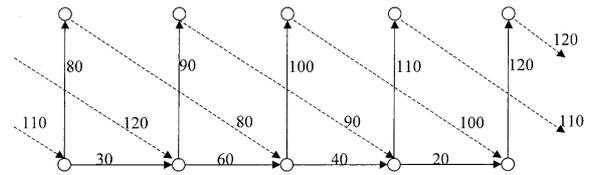


Fig. 5. Optimal solution for Scenario 4.

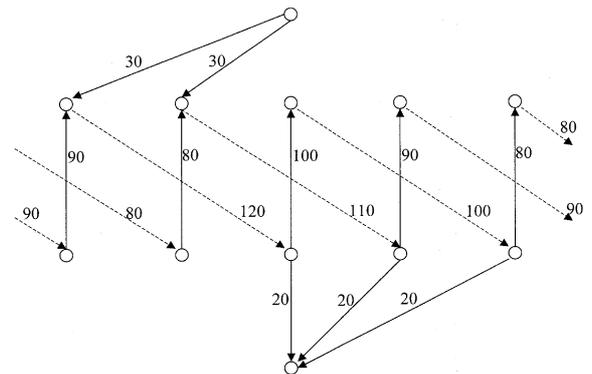


Fig. 6. Optimal solution for Scenario 5.

3. Computational complexity when the costs are concave

For the production planning problem without remanufacturing, very good results have been found when the costs are concave. Most importantly, Wagner and Whitin (1959) gave an $O(T^2)$ solution to a special case in which storage costs are linear and production costs can be partitioned into set-up components and linear components. Recently, Federgruen and Tzur (1991), Wagelmans *et al.* (1992), and Aggarwal and Park (1993) all provided $O(T \log T)$ algorithms for this special case. For the general concave-cost problem, Zangwill (1969) recognized it to be a minimum-cost network flow problem and pointed out that one of its optimal solutions must form a spanning tree in the network. This observation led to an $O(T^2)$ -time algorithm for the problem.

However, when there are capacity limits on production, the problem is NP-hard even when demands are equal and storages are costless (Florian *et al.*, 1980). The PRP should not be significantly easier than the production planning problem with capacity limits, since demand can be partially met by remanufacturing with the restriction that the items being remanufactured must have been produced and used. In the following theorem (which is parallel to Proposition 1 of Florian *et al.* (1980)), we prove the NP-hardness of PRP.

Theorem 1. *The (PRP) is NP-hard for general concave costs.*

Proof. We reduce the decision-problem version of the KNAPSACK problem, which is NP-hard, to PRP. Here

is a description of the problem: given positive integers a_1, \dots, a_n, A , is there a subset $S \subset \{1, \dots, n\}$ such that $\sum_{i \in S} a_i = A$?

Given any instance of the decision-problem version of KNAPSACK, we define an instance of PRP with a fixed item usage time K . In this PRP instance, for time horizon, we have $T = 2n + 2$ and $K = n + 1$. For demand, we have $D_i = 0, \forall i = -n, \dots, 0, D_1 = nA, D_i = a_{i-1}, \forall i = 2, \dots, n + 1,$ and $D_i = A, \forall i = n + 2, \dots, 2n + 2$. For production costs, we have $P_i(x) = 0, \forall i = 1, \dots, n + 1, x = 0, 1, \dots, P_i(0) = 0, \forall i = n + 2, \dots, 2n + 2,$ and $P_i(x) = A + 2, \forall i = n + 2, \dots, 2n + 2, x = 1, 2, \dots$. For new-item storage costs, we have $H_i(y) = 0, \forall i = 1, \dots, 2n + 1, y = 0, 1, \dots$. For remanufacturing costs, we have $R_i(z) = 0, \forall i = 1, \dots, n + 1, z = 0, 1, \dots, R_i(0) = 0, \forall i = n + 2, \dots, 2n + 2, R_{n+2}(z) = 1, \forall z = 1, 2, \dots,$ and $R_i(z) = 1 + (a_{i-n-2} - 1)z/a_{i-n-2}, \forall i = n + 3, \dots, 2n + 2, z = 1, 2, \dots$. For used-item storage costs, we have $W_i(u) = 0, \forall i = 1, \dots, 2n + 1, u = 0, 1, \dots$. For disposal costs, we have $S_i(v) = 0, \forall i = 1, \dots, 2n + 2, v = 0, 1, \dots$. We claim that the decision-problem version of KNAPSACK has a solution if and only if there exists a feasible production plan for PRP with total cost equal to $A + 1$.

For $t = 1, \dots, n + 1$, all the production and storage costs are zero, so the production plan for the first $n + 1$ periods is arbitrary and it costs zero. Since producing any positive number of items in any period t for $t = n + 2, \dots, 2n + 2$ costs more than satisfying all demands in period t by remanufacturing, and it is actually feasible to satisfy demands in these periods by remanufacturing items demanded in periods $1, \dots, n + 1$, all demands in periods $n + 2, \dots, 2n + 2$ are met by remanufacturing.

We have $D_{n+2} = A > 0$, so $z_{n+2} \geq D_{n+2} > 0$. Due to the fact that $R_{n+2}(z)$ is constantly one and no larger than $R_i(z)$ for any $i = n + 3, \dots, 2n + 2, z = 1, 2, \dots$, and that all storage costs are zero, all nA used items available in period $n + 2$ should be remanufactured to satisfy demands in periods $n + 2, \dots, 2n + 1$. The remanufacturing in periods $n + 3, \dots, 2n + 2$ has to supply the demand of A items in period $2n + 2$. Since for all $i = n + 3, \dots, 2n + 2$, we have

$$R_i(z) = z \quad \text{for } z = 0 \text{ and } z = a_{i-n-2},$$

$$R_i(z) > z \quad \text{for } z = 1, \dots, a_{i-n-2} - 1,$$

the total cost of the production plan is

$$\sum_{i=n+2}^{2n+2} R_i(z_i) = 1 + \sum_{i=1}^n R_{i+n+2}(z_{i+n+2}) \geq 1 + \sum_{i=1}^n z_{i+n+2}$$

$$= 1 + A.$$

The total cost is exactly equal to $A + 1$ if and only if there exists $S \subset \{1, \dots, n\}$ such that $\sum_{i \in S} a_i = A$. When such a set exists, all the used items available in period $i + n + 2$ with $i \in S$ are remanufactured to meet the demand in period $2n + 2$. ■

As a minimum-cost network flow problem, the concave-cost PRP must have a spanning tree as the basis of one of its optimal solutions. A spanning tree is a connected subgraph of the underlying network in which, except for one node, every node has a distinct arc, regardless of its direction, associated with it. The node-arc association is established by applying Euler's node removal procedure for trees. In this procedure, in any stage with at least two nodes left, there are always at least two nodes of degree one. Therefore, we can arbitrarily designate a node in a tree as the only one node not associated with any arc. In PRP, we let this node be O . Each node U_t may be associated with one of four possible arcs: z_t, u_{t-1}, u_t , and v_t . Each node V_t may also be associated with one of four possible arcs: x_t, y_{t-1}, y_t , and z_t . If we consider a spanning tree of the PRP network the combination of T flow patterns with the t th pattern designating the arcs associated with nodes U_t and V_t , we have in total $15 = 4 \times 4 - 1$ possible patterns, where the -1 is due to the common arc z_t .

Thus, the 15 possible patterns for a given period (notice that consecutive periods may have different patterns) are:

- (zx) : U_t with z_t and V_t with x_t ;
- (zy^-) : U_t with z_t and V_t with y_{t-1} ;
- (zy) : U_t with z_t and V_t with y_t ;
- (u^-x) : U_t with u_{t-1} and V_t with x_t ;
- (u^-y^-) : U_t with u_{t-1} and V_t with y_{t-1} ;
- (u^-y) : U_t with u_{t-1} and V_t with y_t ;
- (u^-z) : U_t with u_{t-1} and V_t with z_t ;
- (ux) : U_t with u_t and V_t with x_t ;
- (uy^-) : U_t with u_t and V_t with y_{t-1} ;
- (uy) : U_t with u_t and V_t with y_t ;
- (uz) : U_t with u_t and V_t with z_t ;
- (vx) : U_t with v_t and V_t with x_t ;
- (vy^-) : U_t with v_t and V_t with y_{t-1} ;
- (vy) : U_t with v_t and V_t with y_t ;
- (vz) : U_t with v_t and V_t with z_t .

The patterns above exhaust the possible combinations of positive flow combinations in our problem. For example, the first pattern (zx) implies that both production and remanufacturing occurs at period t ; the second pattern (zy^-) implies that remanufacturing as well as carrying of new items inventory from the previous period occurs at period t , etc.

Note that not every combination of these flow patterns constitute a spanning tree. However, every spanning tree can be partitioned into these flow patterns.

In dealing with the concave-cost production planning problem without remanufacturing, knowing that the basis of one of the optimal solutions is formed by patterns

of (x) and (y^-) for all the periods (In this problem, there is no U_t node, so (x) or (y^-) designates the arc associated with the V_t nodes) leads us to an $O(T^2)$ algorithm with the help of dynamic programming. However, the fact that we are able to identify the solution patterns does not help us obtain a polynomial algorithm for the PRP. A polynomial algorithm is available if certain reasonable conditions hold. However, this development is beyond the scope of the present paper and will not be discussed here.

To understand the above statement, let's focus on (y^-) . In the former case, when the t th pattern is (y^-) , we know for sure that $y_{t-1} - y_t = D_t$; while in the latter case, when the t th pattern contains (y^-) , we have two alternatives: When it is (zy^-) , we have $y_{t-1} - y_t = D_t - B_t$, while when it is (u^-y^-) , (uy^-) , or (vy^-) , we have $y_{t-1} - y_t = D_t$. Therefore, in the former case, the production level in a period with an (x) pattern is solely determined by the number of consecutive (y^-) patterns in the following periods; while in the latter case, the presence of the (zy^-) pattern makes the production level in a period with a pattern including (x) be dependent on the whereabouts of the (zy^-) patterns among the following patterns consecutively including (y^-) , whose possibilities require exponential effort to be exhausted. In light of this observation, knowing that costs are concave offers little help for the PRP.

4. Solution when the costs are linear

When the costs are linear, for any t , there are non-negative constants P_t, H_t, R_t, W_t and S_t , such that $P_t(x_t) = P_t x_t, H_t(y_t) = H_t y_t, R_t(y_t) = R_t y_t, W_t(u_t) = W_t u_t,$ and $S_t(u_t) = S_t u_t$. From the formulation in Section 2, the problem is a linear min-cost flow problem with $O(T)$ nodes and $O(T)$ arcs. The fastest algorithm for this problem that we know of is by Galil and Tardos (1988). For such a problem with n nodes and m arcs, this algorithm runs in $O(n^2(m + n \log n) \log n)$ time. Applying this algorithm directly to our linear-cost PRP takes $O(T^3(\log T)^2)$ time.

According to Wagner (1959), the linear min-cost flow problem with n nodes and m arcs can be transformed into the transportation problem with $O(n^2)$ supply nodes, $O(n)$ demand nodes, and $O(m)$ arcs. The fastest algorithm for the transportation problem is by Kleinschmidt and Schannath (1995). When there are m supply nodes, n demand nodes, and k arcs, the algorithm runs in $O(m \log m(k + n \log n))$ time. So, using Wagner's transformation and Kleinschmidt and Schannath's algorithm in combination, we can solve the aforementioned linear min-cost flow problem in $O(n^2 \log n(m + n \log n))$ time. Hence, we can use this combined method to solve the linear PRP also in $O(T^3(\log T)^2)$ time.

However, the linear-cost PRP can be transformed into the transportation problem in a special way that results in a faster solution algorithm.

We start the transformation from here. Because costs are linear, we can attribute total cost to handling each individual item. We make the following definitions (where $1 \leq t_1 \leq t_2 \leq T$):

- $P_{t_1 t_2}$ = cost of satisfying demand for an item in period t_2 by producing a new item in period t_1 ;
- P_t^* = minimum-cost of satisfying demand for an item in period t by producing a new item;
- τ_t^P = period in which it costs the least to produce a new item to satisfy demand for an item in period t ;
- $R_{t_1 t_2 t_2}$ = cost of satisfying demand for an item in period t_2 by remanufacturing in period t_2 a used item newly available in period t_1 ;
- $R_{t_1 t_2}$ = cost of satisfying demand for an item in period t_2 by remanufacturing a used item newly available in period t_1 ;
- $\tau_{t_1 t_2}^R$ = period in which it cost the least to re-manufacture a used item newly available in period t_1 to satisfy demand for an item in period t_2 ;
- $S_{t_1 t_2}$ = cost of disposing of in period t_2 a used item newly available in period t_1 ;
- S_t^* = minimum-cost of disposing of a used item newly available in period t ;
- τ_t^S = period in which it costs the least to dispose of a used item newly available in period t .

We have, for $t_1 \leq t_2 \leq t_2$,

$$R_{t_1 t_2 t_2} = \sum_{\tau=t_1}^{t_2-1} W_\tau + R_{t_2} + \sum_{\tau=t_2}^{t_2-1} H_\tau.$$

For $t_1 \leq t_2$,

$$P_{t_1 t_2} = P_{t_1} + \sum_{\tau=t_1}^{t_2-1} H_\tau,$$

$$R_{t_1 t_2} = \min_{\tau=t_1}^{t_2} R_{t_1 \tau t_2},$$

$$\tau_{t_1 t_2}^R = \operatorname{argmin}_{\tau=t_1}^{t_2} R_{t_1 \tau t_2},$$

$$S_{t_1 t_2} = \sum_{\tau=t_1}^{t_2-1} W_\tau + S_{t_2}.$$

For any t ,

$$P_t^* = \min_{\tau=1}^t P_{\tau t},$$

$$\tau_t^P = \operatorname{argmin}_{\tau=1}^t P_{\tau t},$$

$$S_t^* = \min_{\tau=t}^T S_{\tau t},$$

$$\tau_t^S = \operatorname{argmin}_{\tau=t}^T S_{\tau t}.$$

For $t_1 > t_2$,

$$P_{t_1 t_2} = R_{t_1 t_2} = S_{t_1 t_2} = +\infty.$$

With the above parameters available, the problem can be represented by another network flow problem, as shown in Fig. 7. In this network, there are $4T + 2$ nodes: One source node, T production nodes, T remanufacturing nodes, T demand nodes, T disposal nodes, and one sink node. If we describe each arc in the network by the triplet (lower bound for the amount of flow, upper bound for the amount of flow; cost of unit flow), we can describe arcs in this network in the following way:

- there is an arc from the source node to each production node t with description $(0, +\infty; 0)$;
- there is an arc from the source node to each remanufacturing node t with description $(B_t, B_t; 0)$;
- there is an arc from each production node t_1 to each demand node t_2 with description $(0, +\infty; P_{t_1 t_2})$;
- there is an arc from each remanufacturing node t_1 to each demand node t_2 with description $(0, +\infty; R_{t_1 t_2})$;
- there is an arc from each remanufacturing node t_1 to each disposal node t_2 with description $(0, +\infty; S_{t_1 t_2})$;
- there is an arc from each demand node t to the sink node with description $(D_t, D_t; 0)$; and,

- there is an arc from each disposal node to the sink node with description $(0, +\infty; 0)$.

The problem equivalent to the original problem on this new network is to find the least costly feasible flow for this network. By the minimization nature of the problem, if there is a positive amount of flow in the arc from production node t_1 to demand node t_2 , it must be that

$$t_1 = \tau_{t_2}^P \leq t_2,$$

and, if there is a positive amount of flow in the arc from remanufacturing node t_1 to disposal node t_2 , it must be true that

$$t_2 = \tau_{t_1}^S \geq t_1.$$

Because it is feasible to send any amount of flow on the production-demand arcs and remanufacturing-disposal arcs, any unit flow from remanufacturing node t_1 to demand node t_2 can be thought of as replacing a unit flow from production node $\tau_{t_2}^P$ to demand node t_2 and a unit flow from remanufacturing node t_1 to disposal node $\tau_{t_1}^S$. The possible saving from this replacement is

$$\Delta_{t_1 t_2} = P_{t_2}^* + S_{t_1}^* - R_{t_1 t_2}.$$

Any $f_{t_1 t_2}$ units of flow from remanufacturing node t_1 to demand node t_2 results in the reduction of additional $f_{t_1 t_2}$ units of flow from remanufacturing node t_1 to disposal node $\tau_{t_1}^S$ and additional $f_{t_1 t_2}$ units of flow from production node $\tau_{t_2}^P$ to demand node t_2 . So, once we know all the $f_{t_1 t_2}$'s, we know the entire optimal flow pattern as con-

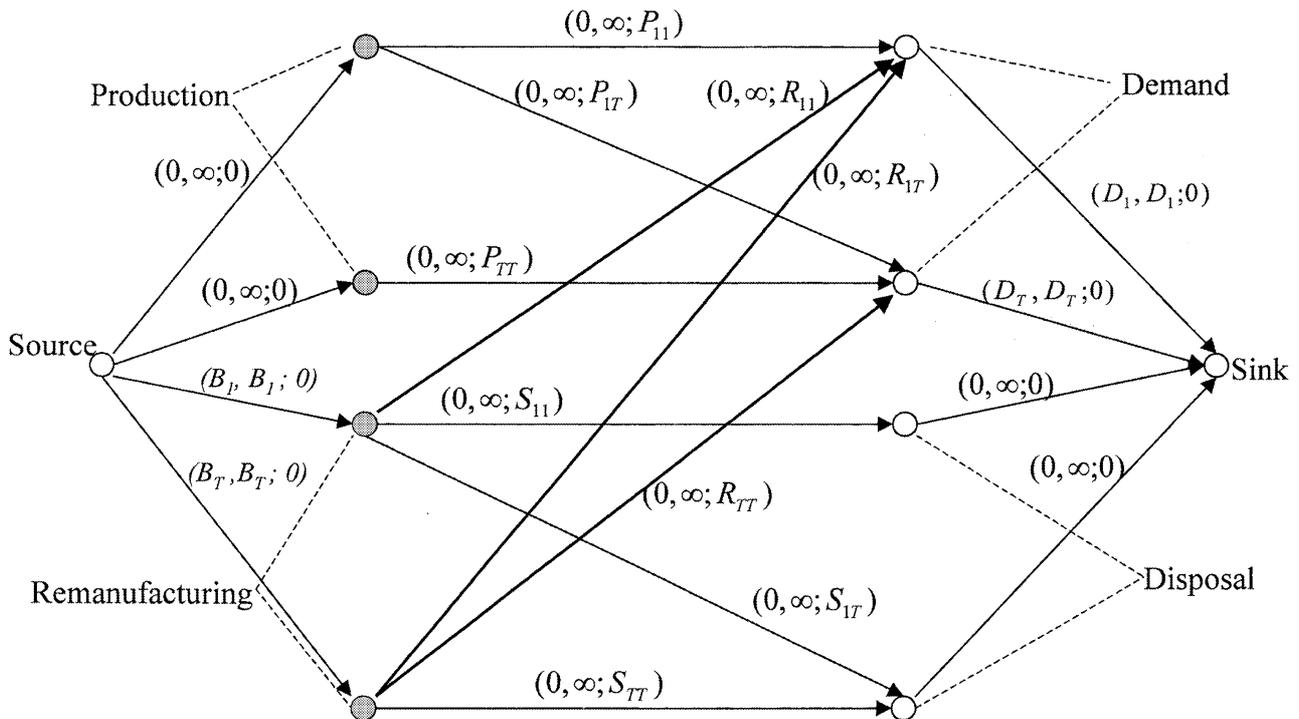


Fig. 7. Another network representation of the linear-cost case.

sisting of $f_t^P = D_t - \sum_{t_1=1}^T f_{t_1 t}$ units of flow from production node τ_t^P to demand node t for each t , $f_t^S = B_t - \sum_{t_2=1}^T f_{t t_2}$ units of flow from remanufacturing node t to disposal node τ_t^S for each t , and $f_{t_1 t_2}$ units of flow from remanufacturing node t_1 to demand node t_2 for each t_1 and each t_2 . And, we know the total cost as

$$\sum_{t=1}^T (P_t^* D_t + S_t^* B_t) - \sum_{t_1=1}^T \sum_{t_2=1}^T \Delta_{t_1 t_2} f_{t_1 t_2}.$$

So, the original problem boils down to the following:

$$(TRP) \max \sum_{t_1=1}^T \sum_{t_2=1}^T \Delta_{t_1 t_2} f_{t_1 t_2},$$

subject to

$$\begin{aligned} \sum_{t_2=1}^T f_{t_1 t_2} &\leq B_{t_1} \quad \forall t_1 = 1, \dots, T, \\ \sum_{t_1=1}^T f_{t_1 t_2} &\leq D_{t_2} \quad \forall t_2 = 1, \dots, T, \\ f_{t_1 t_2} &= 0, 1, 2, \dots \end{aligned}$$

This is the transportation problem with $O(T)$ supply nodes, $O(T)$ demand nodes, and $O(T)$ arcs. Applying Kleinschmidt and Schannath's algorithm, it can be solved in $O(T^2(\log T)^2)$ time. To translate $f_{t_1 t_2}$'s, f_t^P 's, and f_t^S 's back to x_t 's, y_t 's, z_t 's, u_t 's, and v_t 's, we use the following formulae:

$$x_t = \sum_{t_2 \leq t \leq T \text{ such that } \tau_{t_2}^P = t} f_{t_2}^P,$$

$$y_t = \sum_{t+1 \leq t_2 \leq T \text{ such that } \tau_{t_2}^P \leq t} f_{t_2}^P + \sum_{1 \leq t_1 \leq t, t+1 \leq t_2 \leq T \text{ such that } \tau_{t_1 t_2}^R \leq t} f_{t_1 t_2},$$

$$z_t = \sum_{1 \leq t_1 \leq t, t \leq t_2 \leq T \text{ such that } \tau_{t_1 t_2}^R = t} f_{t_1 t_2},$$

$$u_t = \sum_{1 \leq t_1 \leq t \text{ such that } \tau_{t_1}^S \geq t+1} f_{t_1}^S,$$

and

$$v_t = \sum_{1 \leq t_1 \leq t \text{ such that } \tau_{t_1}^S = t} f_{t_1}^S.$$

We may summarize the procedure in the following steps:

- Step 1. Calculate $R_{t_1 t_2}$'s for $1 \leq t_1 \leq t_2 \leq T$.
- Step 2. Calculate $P_{t_1 t_2}$'s, $R_{t_1 t_2}$'s, $\tau_{t_1 t_2}^R$'s, and $S_{t_1 t_2}$'s for $1 \leq t_1 \leq t_2 \leq T$.
- Step 3. Calculate P_t^* 's, τ_t^P 's, S_t^* 's, and τ_t^S 's for $1 \leq t \leq T$.
- Step 4. Calculate $\Delta_{t_1 t_2}$'s for $1 \leq t_1 \leq t_2 \leq T$.
- Step 5. Solve TRP to get $f_{t_1 t_2}$'s.

Step 6. $f_t^P = D_t - \sum_{\tau=1}^P f_{t\tau}$ and $f_t^S = B_t - \sum_{\tau=t}^T f_{t\tau}$ for $t = 1, \dots, T$.

Step 7. Translate $f_{t_1 t_2}$'s, f_t^P 's, and f_t^S 's into x_t 's, y_t 's, z_t 's, u_t 's, and v_t 's.

Calculating $R_{t_1 t_2}$'s and z_t 's takes the bulk $O(T^3)$ time. The complexity of the procedure is hence $O(T^3)$, $O((\log T)^2)$ times faster than through the brute-force transformation. Furthermore, when we have: (I) $H_t \geq W_t$ for any t and thus $\tau_{t_1 t_2}^R = t_2$ for any $t_1 \leq t_2$; or (II) $H_t \leq W_t$ for any t and thus $\tau_{t_1 t_2}^R = t_1$ for any $t_1 \leq t_2$, solving the related transportation problem dominates the effort and the complexity of the procedure reduces to $O(T^2(\log T)^2)$.

5. Final comments

In this paper, we have studied the production planning problem with the option of remanufacturing (PRP). The PRP is much more difficult than the conventional production planning problem without the remanufacturing option. Only for linear-costs cases have we obtained polynomial-time algorithms. For the problem in general, the best we can say is that it can be solved in pseudo-polynomial time through dynamic programming. In future work we plan to explore special conditions that will enable polynomial-time solution procedure for special concave-cost functions and as well as to address convex or arbitrary cost functions. We believe that the way to go there would be to linearize the cost functions and then to use the exact polynomial algorithm to optimally solve the approximate problem.

In this paper, we have not allowed the option of backlogging. Allowing backlogging will make the problem less restrictive and more interesting, especially when production costs are convex and there is a dilemma in choosing between averaging production levels in different periods to save cost and producing as many items as possible in early periods to fully take advantage of the relatively cheap cost of remanufacturing. However, the point where interesting things happen is also the point where trouble brews: the maximal level of remanufacturing from each period is no longer fixed to be the demand in that period, but the full-filled demand up to that period.

Lastly, the assumptions of periodic deterministic demand and deterministic flow of returned items are quite restrictive for practical use. The latter is probably more prone to uncertainty than the former since it combines two types of uncertainty. First, the consumer behavior – willingness to return items – which has similar stochastic attributes to the consumer demand – willingness to purchase new items. Second, the extent to which the returned items can be remanufactured (or fixed). Models and solution methods that allow stochastic demands need to be found. Due to the presence of remanufacturing which connects things in different periods, this task will conceivably be extremely difficult.

Acknowledgements

Gang Yu's research was supported in part by the National Natural Science Foundation of China under grant number 79928001, and by the FADRC grant and the special research grant from the University of Texas at Austin.

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