MODELS FOR IMPROVED EFFECTIVENESS BASED ON DEA EFFICIENCY RESULTS

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Following the characterization via Data Envelopment Analysis (DEA) of managerial units as *efficient* or *inefficient*, management will wish to increase profitability and/or control costs while becoming (or remaining) technically efficient in the DEA sense. This paper presents three families of models for achieving this and describes the managerial situations in which they are useful. The first addresses the management of an existing Decision Making Unit (DMU) and the second attempts to identify the desired "location" for a new DMU. The third addresses the aggregate of all DMUs, reallocating scarce resources among them for maximum overall organizational profitability and technical efficiency.

■ Data Envelopment Analysis (DEA) classifies Decision Making Units (DMUs) as fully (relatively) efficient or less than fully (relatively) efficient, and provides useful efficiency diagnostics ([1], [4], [5]). The DMUs use differing amounts of the same input resources to produce differing amounts of the same productive outputs. This paper begins where DEA leaves off. We consider a central authority which controls the aggregate budget, and sets goals for an organization within which several DMUs are defined. The question addressed by this paper is: how may these goals be defined and the budget allocated in a way conducive to meeting the overall organizational goals, utilizing the results of DEA?

A central feature of DEA is that prior weights on input and output dimensions are neither required nor set. The models presented in this paper involve *effectiveness* goals that set the direction of the enterprise, and require, in general, the introduction of such weights. DEA, in contrast, is concerned with the *efficiency* with which DMUs achieve the objectives stemming from these goals/directions (see the discussion of these terms in [1], p. 71). Elsewhere (e.g., [6] page 40 and [8]) efficiency has been equated with performing current activities as well as possible, and effectiveness with choosing the proper activities. Efficiency is thus conservative both in its original thermodynamic sense and in the present organizational sense. For example, Manufacturing Resource Planning (MRP II) achieves efficiencies by means of a preoccupation with production smoothing. A company's future (i.e., its effectiveness), however, depends on the costs and revenues associated with new technologies, new products, and new markets. The pursuit of any of these can cause interruptions in efficiency.

Traditionally, multiple ratios (added value per employee, return on equity, etc.) have been used to measure efficiency/effectiveness performance. DEA has the virtue of reducing these multiple measures to a single *efficiency* score. However, present research in this area, e.g. [7], shows that effectiveness remains, or at least can remain, multi-objective in nature. Also, some input/output variables may lend themselves easily to weighting by prices and others may not. For these reasons, we present not a single technique, but rather a variety of models, each of them appropriate to a specific scenario. These models do not necessarily provide global optimal solutions. Rather, they improve effectiveness by moving the enterprise as a whole, or its individual units, closer to its goals. This is done while keeping the DMUs technically efficient. In addition, the models provide systematic means for monitoring the DMUs activities.

The two-stage process we employ, starting with the DEA phase and continuing with the models offered here, can occur, for example, in a marketing situation where efficiency is desired in achieving market penetration, market share, and brand loyalty, but where profit remains a unifying criterion [8]. Alternatively, in a nonprofit situation, such as the school system mentioned in [5], multiple output goals may need to be met as closely as possible under a reduction in total budget.

The models developed in this paper are categorized into three families. First, a group of models is suggested to handle effectiveness improvement in an existing DMU. Second, we treat dynamic systems in which the number of entities in the organization varies and new DMUs are to enter the organization. This situation, heretofore not dealt with in the DEA literature, is of great importance when, for example, a manufacturer considers the introduction of a new brand into an existing market. Third, we address the issue of reallocating scarce resources among the aggregate of all DMUs for maximum overall organizational profitability and technical efficiency.

All of these models assume that results from a DEA study are available for the set of relevant DMUs. The

DEA methodology is comprised of several alternative (but related) programs corresponding to alternative notions of efficiency. For the sake of consistency we shall assume throughout this paper that the additive DEA model [2] was used in the first phase. Only minor modifications will be required to use the models with other DEA formulations.

In the next section we introduce notation that remains consistent through the presentation of the several models. Then we present "micro" models addressing single (existing or new) DMUs under different budget and informational scenarios. Following this, we move to a "macro" model which (re)allocates resources across all DMUs simultaneously. The objective of this model is to enhance effectiveness while striving for (or maintaining) maximum efficiency in the DEA sense. A numerical example taken from the literature provides an ongoing illustration of the solution characteristics and implications of these models. A concluding section offers a summary of the management/organizational scenarios to guide the user to the most appropriate of the models presented.

Technical Preliminaries

The notation used in the paper is summarized below.

Indices:

DMUs $j, \ell=1,\ldots,n$

Outputs $r=1,\ldots,k$

Inputs $i=1,\ldots,m$

Distance Measures:

L_1 norm: the sum of the absolute values of the differences between two points

L_2 norm: the square root of the sum of the squared differences between two points.

Data:

y_{rj} = value of the r-th output of DMU j; an entry in Y, the $k \cdot n$ output matrix

x_{ij} = value of i-th input for DMU j; an entry in X, the $m \cdot n$ input matrix

p, c = output and input price/cost vectors, respectively

X_j = input vector for a specific DMU$_j$; $X_j^T = (x_{1j}, x_{2j},\ldots,x_{mj})$

Y_j = output vector for a specific DMUj; $Y_j^T = (y_{1j}, y_{2j},\ldots,y_{kj})$

B = Budget (value of resources allocated to a particular DMU)

R_i = available amount of resource i to be allocated to all the DMUs

π_i = allowed proportional deviation from the i-th input original allocation for each DMU

w^+, w^- = weight parameters in an objective function

E = set of all efficient DMUs

F_j = the set of facet DMUs associated with a particular DMU$_j$

α = fractional change from original budget

U = upper limit on overall proportional deviation of a DMU from its original input-output profile

Variables:

$\sigma_{i\ell}$ = slack value for input i in the evaluation of DMU$_\ell$

$s_{r\ell}$ = slack value for output r in the evaluation of DMU$_\ell$

$\lambda_{j\ell}$ = weight of DMU j in the reference facet of DMU$_\ell$

e_ℓ = the efficiency score of DMU$_\ell$

v_{ij}, u_{ij} = upper and lower deviation variables

δ^-, δ^+ = deviations from required changes in the budget.

In order to illustrate the abstract models given in this paper we use a single numerical example (taken from [3]) with 10 DMUs, 2 inputs and 2 outputs. The example is here expanded to include assumed values of output prices and input costs for the relevant models. Example data are shown in Table 1.

Improving the Effectiveness of An Existing DMU

The models advanced in this section address the movement of a DMU for increased effectiveness within the facet that originally (in the DEA) enveloped it. The fac-

Table 1. Example Data

DMU (j)	Inputs (i)		Outputs (r)	
	1	2	1	2
1	9	9	2	1
2	12	8	3	1
3	7	12	2	2
4	6	10	5	3
5	10	5	4	4
6	8	10	3	3
7	12	10	6	6
8	14	6	8	2
9	12	12	1	6
10	8	8	3	5
Prices	6	2	4	8

et (F_j) for each DMU$_j$ is defined as all the convex combinations of the input-output values associated with the DMUs which determined DMU$_j$'s efficiency in DEA (see [1]). DEA considers the DMU's facet members to be its most appropriate comparison set since these are efficient units whose input-output values closely resemble the evaluated DMU and serve as the yardstick in its efficiency evaluation process. Restricting the movement of a DMU to be within its facet boundaries corrolates with this notion. Furthermore, as "distance" in the input-output space corresponds to behavioral change on the part of the DMU (and managerial reallocation of resources), limiting the DMU's movement to the boundaries of the facet may best conform to realities of organizational change and decentralized control.

The several models presented in this section share a common structure. However, they show that different organizational types (e.g., for profit or not for profit) and goals, and different assumptions and available information (e.g., on costs and benefits), require different modeling considerations within this common structure (see Figure 1).

Model (1) below is the additive model of DEA [2], which is used as our starting point. The additive DEA model evaluates the efficiency (e_ℓ) of each DMU$_\ell$, $\ell = 1,\ldots,n$, as follows:

$$
\begin{aligned}
e_\ell = \text{Max} & \sum_{r=1}^{k} s_{r\ell} + \sum_{i=1}^{m} \sigma_{i\ell} & & (1)\\
\text{s.t.} & \sum_{j=1}^{n} y_{rj}\,\lambda_{j\ell} - s_{r\ell} = y_{r\ell} & r=1,\ldots,k \\
& \sum_{j=1}^{n} x_{ij}\,\lambda_{j\ell} + \sigma_{i\ell} = x_{i\ell} & i=1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_{j\ell} = 1 \\
& \lambda_{j\ell},\ s_{r\ell},\ \sigma_{i\ell} \geq 0
\end{aligned}
$$

Applying program (1) separately to each DMU$_\ell$ of Table 1, using in the constraints the data for all DMUs, reveals DMUs 4, 5, 7, 8 and 10 as fully relatively efficient. For the relatively inefficient DMU$_1$, program (1) also provides a projection onto a section of the efficiency frontier given by the facet $F_1 = \{4, 5, 7, 10\}$ (see Table 2). This projection, determined by optimal values of the "slack" variables s and σ, can be described as the point on the frontier that is farthest from the original location of DMU$_1$, subject to the requirement that the projection must be in a direction of nonincreasing inputs and nondecreasing outputs. Clearly, there is scope

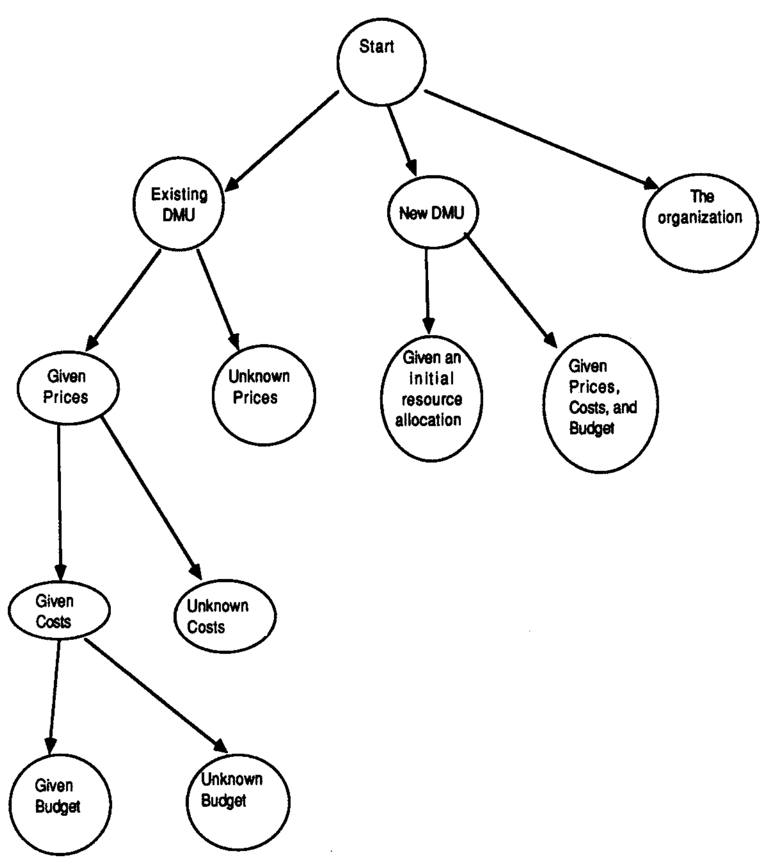

Figure 1: Decision Diagram for the Effectiveness Models

Table 2. Projection of DMU_1 by DEA			
Outputs (r)	Actual Value (y_{r1})	Slack (s_{r1})	Projected Value
1	2	2.25	4.25
2	1	2.75	3.75
Inputs (i)	Actual Value (x_{i1})	Slack (σ_{i1})	Projected Value
1	9	0	9
2	9	2.75	6.25

Optimal lambda (λ_{j1}) values in evaluating DMU_1: $\lambda_{41} = 0.25$, $\lambda_{51} = 0.75$, $\lambda_{71} = \lambda_{10,1} = 0.0$. DMUs (e.g., DMU_7 and DMU_{10}) whose optimal lambda values are zero, but whose corresponding reduced costs are also zero, are considered to be in the facet.

for applying additional criteria (as we shall do in the later models of this section) for moving DMU_1's projection in more effective directions while keeping it on the efficient facet.

In contrast with (1), program (2) determines the shortest distance (using the L_1 or L_2 norms) from a DMU to its facet. From the point of view of overall resource usage and production level attainment, and in the absence of input-output weights, using the sum of all slacks is a reasonable approach to determining changes that need to be undertaken to achieve efficiency.

$$e_\ell = \text{Min} \sum_{r=1}^{k} s_{r\ell} + \sum_{i=1}^{m} \sigma_{i\ell} \qquad \text{(using the } L_1 \text{ norm)} \quad (2)$$

or

$$e_\ell = \text{Min} \left[\sum_{r=1}^{k} s_{r\ell}^2 + \sum_{i=1}^{m} \sigma_{i\ell}^2\right]^{0.5} \qquad \text{(using the } L_2 \text{ norm)}$$

s.t.

$$\sum_{j \in F_\ell} y_{rj}\, \lambda_{j\ell} - s_{r\ell} = y_{r\ell} \qquad r=1,\ldots,k$$

$$\sum_{j \in F_\ell} x_{ij}\, \lambda_{j\ell} + \sigma_{i\ell} = x_{i\ell} \qquad i=1,\ldots,m$$

$$\sum_{j \in F_\ell} \lambda_{j\ell} = 1$$

$$\lambda_{j\ell},\ s_{r\ell},\ \sigma_{i\ell} \geq 0 \quad j \epsilon F_\ell.$$

Applying model (2) to DMU_1 yields the minimum L_1 and L_2 projections shown in Tables 3 and 4, respectively.

Given a vector of prices (or benefits) p, and a vector of input factor costs c, program (3) finds the most effective (i.e., maximum profit or net benefit) projection onto the facet F_ℓ corresponding to an evaluated DMU_ℓ, $\ell = (1,2,\ldots,n)$. We denote the projection as "DMU_Q."

$$\text{Max} \sum_{r=1}^{k} p_r y_{rQ} - \sum_{i=1}^{m} c_i x_{iQ} \qquad (3)$$

s.t.

$$\sum_{j \in F_\ell} y_{rj}\, \lambda_{j\ell} - y_{rQ} = 0 \qquad r=1,\ldots,k$$

$$\sum_{j \in F_\ell} x_{ij}\, \lambda_{j\ell} - x_{iQ} = 0 \qquad i=1,\ldots,m$$

Table 3. Minimum L_1 Projection of DMU_1 onto its Facet			
Outputs (r)	Actual Value (y_{r1})	Slack (s_{r1})	Projected Value
1	2	2.3333	4.3333
2	1	4.0	5
Inputs (i)	Actual Value (x_{i1})	Slack (σ_{i1})	Projected Value
1	9	0.0	9
2	9	0.0	9

Optimal lambda (λ_{j1}) values in evaluating DMU_1: $\lambda_{41} = .166$, $\lambda_{51} = .00$, $\lambda_{71} = .333$, $\lambda_{10,1} = .5$

Table 4. Minimum L_2 Projection of DMU_1 onto its Facet			
Outputs (r)	Actual Value (y_{r1})	Slack (s_{r1})	Projected Value
1	2	2.3333	4.3333
2	1	2.6666	3.6666
Inputs (i)	Actual Value (x_{i1})	Slack (σ_{i1})	Projected Value
1	9	1.3333	7.6666
2	9	1.0	8

Optimal lambda (λ_{j1}) values in evaluating DMU_1: $\lambda_{41} = .5$, $\lambda_{51} = .333$, $\lambda_{71} = .00$, $\lambda_{10,1} = .166$

$$\sum_{j \in F_\ell} \lambda_{j\ell} = 1$$

$$y_{rQ},\ x_{rQ},\ \lambda_{j\ell} \geq 0 \quad j\epsilon F_\ell; \quad \forall r$$

Table 5 illustrates the application of program (3) to DMU_1. If the costs c are not known, they may be set to zero in (3). The solution will then yield the maximum revenue (benefit) position, without regard to cost.

The objective here is to determine the input/output changes $\Delta X_\ell = (\Delta x_{1\ell},\ldots,\Delta x_{m\ell})$, $\Delta Y_\ell = (\Delta y_{1\ell},\ldots,\Delta y_{k\ell})$ which will maximize incremental profit for an existing DMU_ℓ under the same information scenario given earlier and, in addition, with a given fractional change α in its budget.

$$\text{Max} \sum_{r=1}^{k} p_r \Delta y_{r\ell} - w^+\delta^+ + w^-\delta^- \qquad (4)$$

s.t.

$$\sum_{j \in F_\ell} y_{rj}\, \lambda_{j\ell} - s_{r\ell} - \Delta y_{r\ell} = y_{r\ell} \qquad r=1,\ldots,k$$

$$\sum_{j \in F_\ell} x_{ij}\, \lambda_{j\ell} + \sigma_{i\ell} - \Delta x_{i\ell} = x_{i\ell} \qquad i=1,\ldots,m$$

$$\sum_{i=1}^{m} c_i \Delta x_{i\ell} + \delta^+ - \delta^- = \alpha \sum_{i=1}^{m} c_i x_{i\ell}$$

$$\sum_{j \in F_\ell} \lambda_{j\ell} = 1$$

$$\lambda_{j\ell},\ s_{r\ell},\ \sigma_{i\ell},\ \delta^-,\ \delta^+,\ \geq 0;\ \Delta x_{i\ell},\ \Delta y_{r\ell} \text{ unrestricted.}$$

An increase in the budget is implied by $\alpha > 0$, and $\alpha < 0$ implies a decrease in the budget. Given this allowed proportional change in current budget; the current input and output values for the DMU; output prices and input costs; and the set of efficient reference units, this program sets input and output goals to achieve highest profitability for the DMU while becoming (or remaining) as efficient as possible. When $\alpha > 0$, ΔX_ℓ will be nonnegative and the $\sigma_{i\ell}$ will absorb excess input amounts which otherwise would have caused the projection to move beyond the facet F_ℓ. Similarly, on the output side, since ΔY_ℓ has positive coefficients in the objective, output goals will be set as high as allowed by the facet and the $s_{r\ell}$ values will remain at zero. Conversely, for $\alpha < 0$, ΔX_ℓ will be nonpositive and the $s_{r\ell}$ will absorb deviations outside of the facet on the output side. The deviation term δ^- allows spending in excess of the recommended budget. Weighted appropriately in the functional (relative to δ^+, the deviation below the budget) it allows for initiative at lower levels of the organization: an employee may point out that for a few dollars more, disproportionate benefits may be reaped. Of course, the term and its coefficient may be excluded when they are not applicable.

It is interesting to note that in this example, forcing an increase in the inputs led to a smaller increase in revenues than did forcing a decrease in the inputs! This is an excellent example of the fact that improving effectiveness does not always imply an increase in resources but rather, in some cases, just the opposite. Tables 6 and 7 illustrate the application of model (4) to DMU_1 with $\alpha = 0.1, -0.05$ respectively.

Table 5. Maximum "Profit" Projection of DMU_1 onto its Facet

Outputs (r)	Actual Value (y_{r1})	Projected to 4	Projected to 10
1	2	5	3
2	1	3	5
Inputs (i)	**Actual Value (x_{i1})**	**Projected to 4**	**Projected to 10**
1	9	6	8
2	9	10	8
Profit	−56	−12	−12

Optimal lambda (λ_{j1}) values in evaluating DMU_1: $\lambda_{41} = 1.0$, $\lambda_{51} = 0.0$, $\lambda_{71} = 0.0$, $\lambda_{10,1} = 0.0$

Table 6. Maximum Revenue Increase with 10% Budget Increase for DMU_1

Outputs (r)	Actual Value (y_{r1})	Revenue	Projected	Revenue	Revenue Increase
1	2	8	3.085714	12.343	
2	1	8	5.028572	40.229	
	Total	16		52.5714	36.5714
Inputs (i)	**Actual Value (x_{i1})**	**Cost**	**Projected**	**Cost**	**Cost Increase**
1	9	54	9.885714	59.31428	
2	9	18	9.942857	19.88571	
	Total	72		79.2	7.2 (= +10%)

Optimal lambda (λ_{j1}) values in evaluating DMU_1: $\lambda_{41} = \lambda_{51} = .00$, $\lambda_{71} = .029$, $\lambda_{10,1} = .971$

Table 7. Maximum Revenue Increase with 5% Budget Decrease for DMU_1

Outputs (r)	Actual Value (y_{r1})	Revenue	Projected	Revenue	Revenue Increase
1	2	8	4.242857	16.9714	
2	1	8	5.414286	43.3142	
	Total	16		60.2857	44.2857
Inputs (i)	**Actual Value (x_{i1})**	**Cost**	**Projected**	**Cost**	**Cost Increase**
1	9	54	8.342857	50.05714	
2	9	18	9.171429	18.34286	
	Total	72		68.4	3.6 (= −5%)

Optimal lambda (λ_{j1}) values in evaluating DMU_1: $\lambda_{41} = \lambda_{51} = .00$, $\lambda_{71} = .414$, $\lambda_{10,1} = .586$

Finding the Most Effective Location for a New DMU

The dynamic nature of today's typical organization requires frequent changes and re-adjustments in organizational structure which can involve closing down DMUs and creating new ones. In this section we attempt to show how information on the performance of existing DMUs can offer guidelines to management in setting goals for new DMUs.

Consider the introduction of a new DMU, denoted by DMU_0, with a given resource allocation in the form of an input vector X_0. Management wishes to determine output levels y_{ro} ($r = 1,\ldots,k$) that are feasible goals or targets that DMU_0 could strive to attain. Program (5) determines the maximum level of overall output that DMU_0 could achieve from X_0, consistent with DMU_0 being overall as efficient as possible with reference to the performance of the original DMUs. These maximum output levels may then be used as the desired goals for the new DMU.

$$\text{Max} \sum_{r=1}^{k} y_{ro} \tag{5}$$

s.t.

$$\sum_{j=1}^{n} y_{rj}\lambda_{jo} - s_{ro} - y_{ro} = 0, \quad r=1,\ldots,k$$

$$\sum_{j=1}^{n} x_{rj}\lambda_{jo} + \sigma_{io} = x_{io}, \quad i=1,\ldots,m$$

$$\sum_{j=1}^{n} \lambda_{jo} = 1$$

$$\lambda_{jo},\ s_{ro},\ \sigma_{io},\ y_{ro} \geq 0.$$

To illustrate, consider a new DMU_0 with input vector $X_0 = (8.5, 8.5)$. Table 8 shows the most effective (highest output) and overall most efficient location for DMU_0 as given by the optimal solution to (5), and the optimal lambda values indicate that this location is determined by the observed efficient performance of DMUs 4, 5 and 7.

Table 8. Most Effective and Efficient Location for a new DMU

Outputs (r)	Actual (not available)	Efficient Projection (y_{ro})
1	-	4.91666
2	-	3.95
Inputs (i)	**Allocated (x_{io})**	**Efficient Projection (not applicable)**
1	8.5	-
2	8.5	-

Optimal lambda (λ_{jo}) values for DMU_0: $\lambda_{40} = .483$, $\lambda_{50} = .3$, $\lambda_{70} = .217$, $\lambda_{10,0} = .0$

In program (5), we found the most effective position for a new DMU under a fixed resource allocation, i.e. with all input values fixed. Now we move to a situation where only the aggregate budget B for the new DMU_0 is given, and input levels (x_{io}) and output levels (y_{ro}) are to be determined under known costs and prices (benefits). First, DEA is applied to the original DMUs to identify the set E of efficient ones. Then the following goal programming problem is solved:

$$\text{Max} \sum_{r=1}^{k} p_r y_{ro} - w^+ \sum_{r=1}^{k} s_{ro} - w^- \sum_{i=1}^{m} \sigma_{io} \tag{6}$$

s.t.

$$\sum_{j \in E} y_{rj}\lambda_{jo} - s_{ro} - y_{ro} = 0, \quad r=1,\ldots,k$$

$$\sum_{j \in E} x_{ij}\lambda_{jo} + \sigma_{io} - x_{io} = 0, \quad i=1,\ldots,m$$

$$\sum_{i=1}^{m} c_i x_{io} \leq B$$

$$\sum_{j \in E} \lambda_{jo} = 1$$

$$s_{ro},\ \sigma_{io},\ \lambda_{jo} \geq 0, \quad j \in E.$$

The objective function of program (6) maximizes revenue (benefit) for DMU_0 while minimizing the weighted deviation of DMU_0 from the efficient frontier as determined via the slack variables in the constraints. Input levels are further restricted by the budget constraint. Here we allow no deviation over budget and assign a common weight to output slacks and a common weight to input slacks. Generalizations to accommodate expenditures in excess of budget and differential weighting of the slack variables can be had in a straightforward manner.

Thus program (6) determines X_0^*, Y_0^*, the most effective (i.e., profitable) input and output vectors for DMU_0 which are as close as possible to the efficient frontier. The relative magnitudes of the objective function prices and weights determine the trade-off between effectiveness and efficiency. Y_0^* then represents the most profitable target levels for the outputs for the new DMU given the budget B and as efficient as possible use of inputs X_0^*.

To illustrate, consider a new DMU with given budget $B=70$ (by way of comparison the current budget for DMU_1 is $B=72$). Solving program (6) with the same cost and price parameters as before, we obtain the results shown in Table 9. Target output levels (and input usages) for the new DMU are a combination of those achieved by the efficient DMUs 7 and 10, as indicated by $\lambda_{70} = .214$ and $\lambda_{10,0} = .786$. With this input-output profile the new DMU would be most profitable (under budget B) and fully efficient.

Organization-Wide Effectiveness Improvement

We now move to the "macro" level and consider the problem of reallocating resources across the aggre-

Table 9. Maximum Profit Efficient Location of a New DMU with Given Budget		
Outputs (r)	Actual (not applicable)	Efficient Projection
1	-	3.6428
2	-	5.214
Inputs (i)	Actual (not applicable)	Efficient Projection
1	-	8.857
2	-	8.428
Optimal lambda (λ_{j0}) values for DMU$_0$: $\lambda_{40} = \lambda_{50} = .00$, $\lambda_{70} = .214$, $\lambda_{10,0} = .786$		

gate of DMUs. X_{ij} denotes the initial allocation and x_{ij} denotes the reallocation of input i to DMU$_j$. Resource availabilities R_i are accommodated for individual input dimensions i, as are bounds π_i on the proportional change allowed in input i. A bound U may be placed on the proportional change in the overall position of each DMU (i.e., on the sum of its proportional changes across all input dimensions).

The reallocation involves a five-step computation.

1. Run DEA to compute efficiency scores $e_j, j=1,\ldots,n$. Since in the additive model (used for the underlying DEA analysis) efficiency is associated with the sum of slacks, it was necessary to transform these sums into an efficiency measure. The transformation used here was exp(−(sum of slacks)/(sum of inputs and outputs)). This transformation yields a score of 1.0 for efficient DMUs and a number in the interval (0,1) for inefficient ones.
2. For each DMU$_j$ compute $m \cdot k$ *effectiveness indices*.
$$Z_{irj} = Y_{rj}/X_{ij},\ r=1,\ldots,k,\ i=1,\ldots,m$$
3. Scale the indices by their averages across the n DMUs
$$\xi_{irj} = Z_{irj}/Z_{ir}$$
where
$$Z_{ir} = (1/n)\sum_{j=1}^{n} Z_{irj}.$$
4. For each DMU$_j$ compute m *input effectiveness indices*:
$$\beta_{ij} = (1/k)\sum_{r=1}^{k} \xi_{irj}.$$
5. Solve the following program
$$\text{Max} \sum_{i=1}^{m}\sum_{j=1}^{n} \beta_{ij} e_j x_{ij} \quad (7)$$
$$\begin{aligned} \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \le R_i, && i=1,\ldots,m \\ & x_{ij} \le (1+\pi_i) X_{ij} && \forall i,j \\ & x_{ij} \ge (1-\pi_i) X_{ij} && \forall i,j \\ & (1/X_{ij}) x_{ij} + v_{ij} - u_{ij} = 1 && \forall i,j \\ & \sum_{i=1}^{m} v_{ij} \le U, && j=1,\ldots,n \\ & \sum_{i=1}^{m} u_{ij} \le U, && j=1,\ldots,n \\ & x_{ij},\ v_{ij},\ u_{ij} \ge 0. && i=1,\ldots,m,\ j=1,\ldots,n \end{aligned}$$

The objective function of program (7) uses the efficiency scores e_j to weigh the effectiveness indices β_{ij} in guiding the new allocation of resources. Thus the optimal reallocation X^*_{ij} is maximally effective subject to resource availabilities R_i, allowable proportionate changes π_i in each input dimension for each DMU, and allowable proportionate change U for each DMU as a whole. Table 10 shows the values for the effectiveness indices for our numerical example.

First, we use program (7) to reallocate the initial amounts of resources that were available (i.e., $R_1=98$, $R_2=90$), with bounds of 10% on the proportional change allowed in each input for each DMU (i.e., $\pi_i = 0.1$, $i=1, 2$) and 5% maximum allowable overall proportional change in the DMU position (i.e., $U = 0.05$). The results are summarized in Table 11. In most cases

Table 10. Effectiveness Indices

DMU	Z_{irj}				ξ_{irj}				β_{ij}	
	Y_1/X_1	Y_1/X_2	Y_2/X_1	Y_2/X_2	Y_1/X_1	Y_1/X_2	Y_2/X_1	Y_2/X_2	X_1	X_1
1	.222	.222	.111	.111	.57	.47	.31	.29	.44	.38
2	.25	.375	.083	.125	.64	.79	.24	.32	.44	.55
3	.286	.166	.286	.166	.73	.35	.81	.43	.77	.39
4	.833	.5	.5	.3	2.14	1.05	1.42	.78	1.78	.91
5	.4	.8	.4	.8	1.03	1.68	1.13	2.07	1.08	1.87
6	.375	.3	.375	.3	.96	.63	1.06	.78	1.01	.70
7	.5	.6	.5	.6	1.28	1.26	1.42	2.01	1.35	1.63
8	.571	1.33	.143	.333	1.47	2.81	.40	.86	.93	1.83
9	.083	.083	.5	.5	.21	.17	1.42	1.29	.81	.73
10	.375	.375	.625	.625	.96	.79	1.77	1.62	1.36	1.20
Z_{ir}	.389	.475	.353	.386						

the efficient DMUs were allocated additional resources which were taken in similar proportions from the inefficient DMUs. However, in some cases, e.g., DMU_1 below, inefficient DMUs gained additional resource in one dimension while losing in another, or, as in the case of DMU_9, exhibited small gains in both dimensions.

Next, we allocate a slightly larger resource vector ($R = (105, 95)$) under the same restrictions to see where the inefficient DMUs begin to gain additional resources in spite of their apparent inefficiency. The results in Table 12 show that all the DMUs are now allocated at least as much of the resources as they originally "owned." However, the inefficient units gained only very small relative amounts and the biggest winner was DMU_7 which exhibited the best effectiveness indices in Table 10.

Table 11. Reallocation of Original Resource Vector

		Original		Reallocated	
DMU	Efficiency	X_1	X_2	X_1	X_2
1	.69	9	9	9.009	8.77
2	.687	12	8	11.4	8.0
3	.73	7	12	7.003	11.72
4	1.0	6	10	6.04	10.0
5	1.0	10	5	10.0	5.0
6	.83	8	10	8.0	10.0
7	1.0	12	10	12.53	10.5
8	1.0	14	6	14.005	6.0024
9	.798	12	12	12.0048	12.0048
10	1.0	8	8	8.0	8.0

Table 12. Allocation of an Increased Resource Vector

		Original		Allocated	
DMU	Efficiency	X_1	X_2	X_1	X_2
1	.69	9	9	9.009	9.009
2	.687	12	8	12.005	8.0
3	.73	7	12	7.003	12.005
4	1.0	6	10	6.0024	10
5	1.0	10	5	10	5
6	.83	8	10	8	10
7	1.0	12	10	12.605	10.5
8	1.0	14	6	14.005	6.0025
9	.798	12	12	12.005	12.005
10	1.0	8	8	8	8

Concluding Remarks

In this paper we have extended the DEA efficiency structure to encompass effectiveness models in order to analyze productivity as a combination of efficiency and effectiveness. We presented three situational models: profit maximization for an existing DMU by goal programming; profit maximization for a new DMU by goal programming; and aggregate resource allocation by linear programming. The models correspond to different organizational application scenarios with different information assumed available for each.

A concise, diagrammatic decision guide in which the various models are organized into an overall framework is presented in Figure 1.

Throughout this paper we rely on outcomes of the DEA methodology, and first among these is the notion of a given facet for the evaluated DMU_ℓ. Two reservations should be mentioned in this respect. First, identifying the facet may be a difficult task and, in many cases, the facet is not unique for a given inefficient DMU (see [3]). Second, the facet is selected (as in (1)) according to a mathematical program which attempts to find the worst possible efficiency evaluation for DMU_ℓ. It might be argued here that this selection may be irrelevant from the managerial point of view. However, techniques such as "Constrained Facet Analysis" mentioned in [9] can ensure that the selection will conform to managerial guidelines.

Directions for future research include improving our understanding of the overall productivity of the DMUs. Considering productivity as a function of both efficiency and effectiveness, we need to explore this 'function' and learn more about the possible trade-offs between its components. Collecting empirical data and improving the models, based on experience with the data, will contribute to that end.

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