Inducing Coordination in Supply Chains through Linear Reward Schemes

Boaz Golany, Uriel G. Rothblum

Faculty of Industrial Engineering and Management, Technion–Israel Institute of Technology, Haifa 32000, Israel

Received 28 December 2004; revised 12 May 2005; accepted 18 July 2005
DOI 10.1002/nav.20117
Published online 1 November 2005 in Wiley InterScience (www.interscience.wiley.com).

Abstract: Decentralized decision-making in supply chain management is quite common, and often inevitable, due to the magnitude of the chain, its geographical dispersion, and the number of agents that play a role in it. But, decentralized decision-making is known to result in inefficient Nash equilibrium outcomes, and optimal outcomes that maximize the sum of the utilities of all agents need not be Nash equilibria. In this paper we demonstrate through several examples of supply chain models how linear reward/penalty schemes can be implemented so that a given optimal solution becomes a Nash equilibrium. The examples represent both vertical and horizontal coordination issues. The techniques we employ build on a general framework for the use of linear reward/penalty schemes to induce stability in given optimal solutions and should be useful to other multi-agent operations management settings. © 2005 Wiley Periodicals, Inc. Naval Research Logistics 53: 1–15, 2006.

Keywords: supply chain management; Nash equilibrium; linear reward schemes

1. INTRODUCTION

Responding to the ever-increasing pressures of global markets, supply chains and distribution systems are becoming more and more complex. Inefficiency in managing these systems can result in substantial accumulation of costs that may be critical to the survival of the relevant organizations as global competition puts pressure on profit margins. Considering this reality from a positive perspective, successful management of complex supply chains can be a very important success factor in such organizations. Not surprisingly, the study of supply chains—their characteristics, problems, and potential remedies—has accelerated dramatically during the past decade (e.g., [11, 12, 31, 32, 34]).

A major issue that is being addressed in the practice and literature of supply chains concerns the coordination among the various entities in the chain and the potential negative impact that uncoordinated behavior may have. Of course, the ultimate way of achieving coordination is through centralized control, which addresses all the implications that decisions and their resultant actions may have on an organization. No such perspective is present when decentralized control is practiced and therefore an inherent inefficiency is expected. Indeed, the inefficiency of decentralized decision-making has been well known for a very long time, e.g., the celebrated "prisoner's dilemma" or the "tragedy of the commons" (see [25] and [14], respectively). But, decentralized decision-making is often inevitable, due to the magnitude of the chain, its geographical dispersion, and the number of agents that play a role in it. Furthermore, letting agents pursue what they perceive to be good actions may increase their motivation and identification with the mission of the organization. These behavioral benefits may have an important impact on the organization and its long-term goals.

Formally, in a centralized management system, the centralized decision-maker is looking for actions so as to maximize some global objective function of the organization. In a decentralized organization, the actions of each of numerous agents reflect only their personal preferences, expressed by their individual utility functions. A Nash equilibrium is then the natural outcome in such an environment. We note that the agents' payoffs at a Nash equilibrium are not necessarily unique; further, there need not be even a unique value for the sum of the agents' utilities associated with Nash equilibrium solutions (the sum of the agents' utility is frequently the global utility of the organization). For a thorough review of how these, and other game theoretical concepts, may be applied in the context of supply chain analysis the reader is referred to Cachon and Netessine [8].

The issue of comparing centralized and decentralized decision-making in the context of supply chains and the need to quantify the benefits that might be gained from the

Correspondence to: B. Golany (golany@ie.technion.ac.il)

former have been recently addressed by several authors. For example, Lee, Padmanabhan, and Whang [17] discuss the "bullwhip" phenomenon and its possible negative effects; Chen et al. [10] investigate the impact of forecasting and information-sharing practices on the performance of supply chains; Taylor [33] compares per unit rebates and target rebates to agents to overcome the inefficiency of decentralized control; Cachon [5] reviews various coordination schemes and discusses their properties; Munson, Hu, and Rosenblatt [22] describe a set of numerical examples (aimed towards classroom usage) of such differences in various supply chain settings; Netessine and Rudi [23] develop analytically tractable solutions to centralized and decentralized control of systems with substitutable products.

Further studies have generated a number of instruments aimed at enhancing or imposing coordination among entities in the chain. One such approach, the quick response (e.g., [30]), has been in the area of information sharing. But, a potential problem with information sharing is that the transmission of real-time information may be complex and may have negative side-effects (e.g., leaking confidential information to competitors). Another approach (e.g., flexible contracts [1], vendor managed inventory [26]), attempts to regulate relations among entities in the chain. Other approaches (e.g., gain-sharing mechanisms [4], channel rebates [33]) suggest quantitative incentive models of rewards and penalties to enhance coordination. Cachon [4] proposed a gain-sharing mechanism but refrained from identifying the best proportional allocation of the joint profit or from providing a complete framework for explaining the mechanism he described. The instrument suggested by Taylor [33] is particularly relevant to the approach we discuss here. A channel rebate is a payment from a manufacturer to a retailer based on the retailer's sales to end consumers. Channel rebates are quite common in some industries (e.g., personal computers and peripherals, automotive) where they are usually offered either as "linear rebates" (based on the actual quantity sold by the retailer) or as "target rebates" (based on the sales above a certain target level). But, channel rebates are customarily used to encourage retailers to modify (increase) demand so as to profit the supplier. Our concern is with the global performance of the system. Hence, in our context (e.g., in the example given in Section 3.4) we use channel rebates to induce retailers to act in a globally optimal manner, improving the total gain (rather than benefiting the supplier).

The approach of pricing agents' actions has also been suggested in other contexts such as queueing systems (see, for example, [21]) and transportation systems [2, 13]. Cachon and Zipkin [9] explore the gap between the performance at a Nash equilibrium and optimal performance in a two-stage supply chain. They suggest a mechanism that converts the optimal solution into a Nash equilibrium by transfers that are linear in observed variables (like inventory levels and back orders).

The approaches described above provide ad hoc pricing mechanisms; they are useful and were shown to improve the supply chain performance of various organizations. Still, the literature to date lacks a general approach and a solid foundation of the extent and scope for incentive schemes. The goal of the current paper is to introduce a general framework for the use of incentive mechanisms and demonstrate its usefulness by examples. Specifically, we show how linear rewards (and penalties) on actions of the decision-makers in a supply chain system can be used as incentives that facilitate "closing the gap" between global optimality achieved through overall coordination and Nash equilibria. The "per unit action" rewards we use as incentives are determined by the marginal influence of agents' actions on the welfare of the other agents, evaluated at a given optimal solution (expressible by an explicit formula). We demonstrate that the given optimal solution becomes a (unique) Nash equilibrium under this incentive mechanism. The broad applicability of the formula that expresses the per unit action and, in particular, its potential usefulness in the context of supply chain management, constitutes the main contribution of our paper. We expect that the framework we set for the use of linear prices will be a basis for further studies of mechanisms that induce coordination.

In the economics literature, cross influences of agents' actions on the welfare of other agents are referred to as externalities. The use of marginal externalities as incentives to induce stability of globally desirable outcomes has a long history, for example, see Lindahl [19], Pigou [24], Samuelson [29], and Varian [35]. This literature focuses on specific structures (e.g., public goods, particular taxation) and on the relations of Pareto-optimality and general equilibrium. Our formal framework follows Rothblum [27], who identifies sufficient conditions for the use of marginal externalities to convert optimal performance into a Nash equilibrium. But, some of the examples we present do not satisfy those conditions. In particular, we handle nonconvex cost functions (in the shortage gaming example), discontinuity (in the location coordination example), and actions that are functions of the other agents' actions (in the retailer–supplier price-order coordination example). Further, none of the above references addresses the uniqueness of resulting Nash equilibria. Our analysis of the examples does not logically depend on any of the above references—we verify the effectiveness of the pricing mechanism and the uniqueness of the resulting Nash equilibrium directly (uniqueness is verified in all of our examples but one).

We present the general framework in Section 2 and implement it to four examples in Section 3. To focus on the implementation itself and save the space required for a comprehensive presentation of each example, we deliberately selected known examples where the inefficiency in uncoordinated behavior has already been discussed. Conclusions and directions for further research are included in Section 4.

2. LINEAR REWARD SCHEMES

Consider a supply chain system in which n agents (say, suppliers, wholesalers, distributors, retailers, etc.) operate. Each agent i needs to select action a^i of a set $A^i \subseteq R^{d^i}$ of feasible actions for that agent where actions can be multivariate, that is, $d^i \geq 1$. We assume that each A^i is continuous and determined by functions $g^i_j : R^{d^i} \rightarrow R$ for $j = 1, \ldots, k^i$ so that $A^i = \{x \in R^{d^i} : g^i_j(x) \geq 0 \text{ for } j = 1, \ldots, k^i\}$. When the agents select actions $a^1, \ldots, a^n$, respectively, each agent i gains utility $u^i(a^1, \ldots, a^n)$ where $u^i : \Pi_i A^i \equiv A^1 \times \cdots \times A^n \rightarrow R$ is the utility function of agent i. Thus, the utility gained by each agent depends on the actions taken by *all* agents and not only on his own action; such utility functions are common in many supply chain systems and other situations of multi-agent decision-making. We define the global utility function u to be the sum of the individual agents' utility functions, that is, $u : \Pi_i A^i \rightarrow R$ with $u(a^1, \ldots, a^n) = \sum_{i=1}^n u^i(a^1, \ldots, a^n)$ for each $(a^1, \ldots, a^n) \in \Pi_i A^i$. Finally, we assume perfect information—that is, all the terms defined above are known to all agents.

There are two main approaches to analyze systems with the structure outlined above—the centralized and the decentralized approaches. Under the centralized approach, the agents, or a centralized decision-maker, are to select feasible actions $\bar{a}^1, \ldots, \bar{a}^n$, so as to maximize $u = \sum_{i=1}^n u^i$ over $\Pi_i A^i$ ($\subseteq R\ \Sigma_i\ d^i$). We refer to such $(\bar{a}^1, \ldots, \bar{a}^n)$ as a globally optimal solution. Under the decentralized approach, feasible actions $(\hat{a}^1, \ldots, \hat{a}^n)$ are to be selected by the agents so that for each i, $\hat{a}^i$ maximizes $u^i(\hat{a}^1, \ldots, \hat{a}^{i-1}, a^i, \hat{a}^{i+1}, \ldots, \hat{a}^n)$ over $a^i \in A^i$. We refer to such $(\hat{a}^1, \ldots, \hat{a}^n)$ as a Nash equilibrium solution. It is well known that Nash equilibrium solutions need not be efficient, let alone globally optimal, as is demonstrated by the well-known example of the prisoner's dilemma. Of course, any Nash equilibrium solution is (weakly) dominated by any globally optimal solution.

A globally optimal solution $(\bar{a}^1, \ldots, \bar{a}^n)$ is a solution of the following optimization problem:

$$\max \quad u(a^1, \ldots, a^n) = \sum_{i=1}^n u^i(a^1, \ldots, a^n)$$

$$\text{subject to:} \quad g^i_j(a^i) \geq 0 \text{ for } i = 1, \ldots, n \text{ and } j = 1, \ldots, k^i. \quad (1)$$

Determining Nash equilibrium solutions requires the simultaneous solution of n optimization problems, where the data for each problem is determined by the vectors that constitute solutions of the other problems. So, for $(\hat{a}^1, \ldots, \hat{a}^n)$ to form a Nash equilibrium requires that for each i, $\hat{a}^i$ is a solution of the following optimization problem:

$$\max \quad u^i(\hat{a}^1, \ldots, \hat{a}^{i-1}, a^i, \hat{a}^{i+1}, \ldots, \hat{a}^n)$$

$$\text{subject to:} \quad g^i_j(a^i) \geq 0 \text{ for } j = 1, \ldots, k^i. \quad (2)$$

We define a linear reward scheme to be a set of vectors $c^1 \in R^{d^1}, \ldots, c^n \in R^{d^n}$ along with the interpretation that the utility function of each agent i is modified by adding a (linear) term $(c^i)^T a^i$. The corresponding modified utility function for agent i is then given by $u^i(\cdot|c^i) : \Pi_i A^i \rightarrow R$ where

$$u^i(a^1, \ldots, a^n|c^i) = u^i(a^1, \ldots, a^n) + (c^i)^T a^i \quad \text{for each } (a^1, \ldots, a^n) \in \Pi_i A^i. \quad (3)$$

Of course, negative values of terms $c^i_s a^i_s$ correspond to penalties; hence, it is more accurate to refer to (linear) reward/penalty schemes or to incentive schemes.

Rothblum ([27], Theorem 1) provides conditions under which a given globally optimal solution $\bar{a}^1, \ldots, \bar{a}^n$ is a Nash equilibrium solution under the modified utility functions given by (3) with

$$\bar{c}^i \equiv \left[\sum_{\substack{t=1 \\ t \neq i}}^n \nabla_{a^i} u^t(a^1, \ldots, a^n) \right] \Bigg|_{(a^1, \ldots, a^n) = (\bar{a}^1, \ldots, \bar{a}^n)} \quad \text{for } i = 1, \ldots, n; \quad (4)$$

besides (continuous) differentiability and regularity, the conditions require that each u^i is concave in the a^i variables. Essentially, Rothblum [27] demonstrates that when one uses the right-hand side of (4) as the c^i's within (3), first-order conditions for equilibrium coincide with first-order conditions for global optimality for the original problem. Thus, when the former are sufficient (for Nash equilibrium) and the latter are necessary (for global optimality), the underlying optimal solution becomes a Nash equilibrium. Herein we demonstrate the effectiveness of (4) to induce equilibrium under broader circumstances.

Under the linear reward scheme expressed in (3), agents are to be rewarded for the actions they select on an "action-unit" basis. The right-hand side of (4) sets the action-unit compensation for agent i as the sum of the marginal benefits of his actions on all other agents, evaluated at the given globally optimal solution.

The behavior of the agents is invariant when constant terms are added to the individual utility functions, and such changes have no effect on the set of Nash equilibria solutions. But, constant terms may play an important practical role in the redistribution of profits (or individual costs) that may result from switching to a globally optimal solution since they can be used to provide agents with personal incentives for the move to the new Nash equilibrium. In particular, a given globally optimal solution $\bar{a}^1, \ldots, \bar{a}^n$ is a Nash equilibrium under the modified utility functions given by (3) *if and only if* it is a Nash equilibrium under the utility functions

$$u^i(a^1, \ldots, a^n) + (c^i)^T(a^i - \bar{a}^i) \quad \text{for each } (a^1, \ldots, a^n) \in \Pi_i A^i. \quad (5)$$

The utility functions of (5) have the advantage that the corresponding Nash equilibrium solutions will not require any extra resources that are not used when the actions $\bar{a}^1, \ldots, \bar{a}^n$ are taken under the original utility functions. As the total revenues under the global optimal solution exceed those that are generated at any Nash equilibrium (or any arbitrary feasible solution), an implementation of (5) will generate extra funds that can be used to benefit all agents. The distribution of these excess funds is beyond the scope of the current paper. Interested readers are referred to the vast literature that exists on bargaining games (e.g., [28]) ultimatum and dictator games (e.g., [3]), and related issues in experimental economics (e.g., [16]).

Linear reward schemes that are based on (4) determine individual incentives for the agents. This is the case even when agents' action sets coincide. We say that the system is symmetric and additive if for each agent i, d^i and A^i are independent of i and if $u^i(a^1, \ldots, a^n) = f(a^i, \sum_{t=1}^n a^t)$ where $f: A \times S \to R$ with $S \equiv \{\sum_{t=1}^n a^t : a^t \in A^t \text{ for } t = 1, \ldots, n\}$; in such cases, concavity and the existence of a globally optimal solution assure the existence of a symmetric optimal solution and the corresponding c^i's defined in (4) are invariant of i (e.g., Theorem 2 of [27]). This formulation captures models where the utilities of the agents depend on the actions of the other agents through the average of their actions (e.g., average utilization or consumption of resources, average quality); in such cases, the division of $\sum_{i=1}^n a_i$ by the number of agents n is absorbed in the function f. Further, if J is a subset of agents, such that for each $i \in J$, d^i and A^i are independent of $i \in J$ and $u^i(a^1, \ldots, a^n)$ depends on $\{a^t : t \in J\backslash\{i\}\}$ only through $\sum_{t \in J\backslash\{i\}} a^t$, then concavity of u^i assures the existence of globally optimal solution $\bar{a} = (\bar{a}_1, \ldots, \bar{a}_n)$ with $\bar{a}_i$ invariant of $i \in J$; $\bar{c}_i$ defined by (4) is then invariant of $i \in J$ as well.

In the forthcoming examples, we apply the reward/penalty of (4) to globally optimal solutions and verify that under modifications of the utility functions as given by (3) (and (5)), the given optimal solutions are Nash equilibria; further, all of these solutions are unique Nash equilibria.

3. SUPPLY CHAIN COORDINATION EXAMPLES

This section presents several examples of standard supply chain models in which coordination is beneficial and to which the framework given in the previous section applies. The first three examples describe interactions among retailers about location, replenishment orders, and supplier selection, respectively. The fourth example concerns classical coordination between a retailer and a supplier over wholesale price and order quantity. The last example belongs to a class of "vertical coordination problems" while the other three examples belong to a class of "horizontal coordination problems." Together, these examples demonstrate a broad spectrum of supply chain models to which our approach can be useful.

In all four examples we have $d^i = 1$, that is, each agent's action concerns the selection of a single parameter. Consequently, the subscripts indexing the specific distinct types of the actions of each agent are superfluous. Also, following the tradition in operations management, we use subscripts to index the agents (rather than the superscripts that we used in Section 2).

3.1. Retailers' Locations Coordination

3.1.1. Description of the Problem

We adopt, with minor changes, a standard model that concerns a horizontal coordination problem; see Munson, Hu, and Rosenblatt [22], among others. The lack of coordination in this model and its inefficiency have been substantiated in many case studies. Specifically, consider an ice-cream chain that wishes to operate two ice-cream carts along a two-mile walkway. The continuous demand D for ice cream of each potential customer is assumed to be a decreasing linear function of the distance to the closest cart and is assumed to be given by $D = \beta - d$, where $\beta \geq 2$ is a constant and d is the distance to the nearest cart (if the distance to the two carts is equal, a coin is flipped to determine which cart will be selected). The potential customers are assumed to be spread uniformly along the walkway, and we approximate their number and location with a uniform continuous distribution normalized so that the number of customers in an interval equals its length. We index points along the walkway by their distance from the center point (positive points are located north of the center while negative points are south of it). The locations of the two carts are denoted by a and b with $-1 \leq a, b \leq 1$.

We note that our development extends to cases where the demand function is $\beta - \gamma d$ with $\beta \geq \gamma \geq 0$ and with the distribution of the number of potential customers more general than the uniform one.

3.1.2. *The Cart Operators' Profit Functions*

Denote the expected profit for the two operators when the carts are stationed at locations a and b by $u_1(a, b)$ and $u_2(a, b)$, respectively. If $-1 \leq a < b \leq 1$ then

$$u_1(a, b) = \int_{-1}^{a} [\beta - (a - x)]\, dx + \int_{a}^{(a+b)/2} [\beta - (x - a)]\, dx = \beta\left(\frac{a + b}{2} + 1\right) - \frac{(a + 1)^2}{2} - \frac{(b - a)^2}{8} \quad (6)$$

and

$$u_2(a, b) = u_1(-b, -a) = \beta\left(1 - \frac{a + b}{2}\right) - \frac{(b - a)^2}{8} - \frac{(1 - b)^2}{2}. \quad (7)$$

Evidently, $u_1(a, b)$ and $u_2(a, b)$ are both strictly concave in (a, b). Also, symmetry shows that when $-1 \leq b < a \leq 1$, $u_1(a, b)$ and $u_2(a, b)$ are derivable from (6) and (7) by exchanging the roles of the two operators. Note that values $a = b \neq 0$ form discontinuity points of u_1 and u_2 (as one of the operator's potential customer pools drops while the other's jumps when they switch sides); but, $a = b = 0$ is a continuity point of u_1 and u_2.

3.1.3. *Globally Optimal and Nash Equilibrium Solutions*

As the problem is symmetric, without loss of generality, we restrict attention to the case where $-1 \leq a < b \leq 1$. The global expected profit function is given by

$$u(a, b) = u_1(a, b) + u_2(a, b) = 2\beta - \frac{(a + 1)^2}{2} - \frac{(1 - b)^2}{2} - \frac{(b - a)^2}{4}. \quad (8)$$

We note that u is continuous and strictly concave (jointly) in (a, b) and $(\bar{a}, \bar{b}) = (-\frac{1}{2}, \frac{1}{2})$ is the unique globally optimal solution of the system satisfying $-1 \leq a < b \leq 1$. Of course, symmetry assures that $(\bar{a}, \bar{b}) = (\frac{1}{2}, -\frac{1}{2})$ is another optimal solution.

We note that the extension of u to the region $-1 \leq a = b \leq 1$ will preserve (8) and will have u continuous and concave (although, as noted above, u_1 and u_2 are not necessarily continuous). But, no globally optimal solutions are in the region $-1 \leq a = b \leq 1$. Finally, standard results show that (0, 0) is a unique Nash equilibrium solution with payoff $\beta - \frac{1}{2}$ for each retailer.

3.1.4. *Implementation of the Linear Reward Scheme*

Consider the globally optimal solution $(\bar{a}, \bar{b}) = (-\frac{1}{2}, \frac{1}{2})$. In order to set up the linear reward scheme determined by (3) and (4), consider

$$\bar{c}_1 \equiv \frac{\partial u_2(a, b)}{\partial a}\bigg|_{a=-0.5,b=0.5} = \left[\frac{-\beta}{2} - \frac{a}{4} + \frac{b}{4}\right]\bigg|_{a=-0.5,b=0.5} = \frac{-\beta}{2} + \frac{1}{4} \quad (9)$$

and

$$\bar{c}_2 \equiv \frac{\partial u_1(a, b)}{\partial b}\bigg|_{a=-0.5,b=0.5} = \left[\frac{\beta}{2} - \frac{b}{4} + \frac{a}{4}\right]\bigg|_{a=-0.5,b=0.5} = \frac{\beta}{2} - \frac{1}{4}. \quad (10)$$

Using $\bar{c}_1$ and $\bar{c}_2$ as per-unit rewards for the locations the operators select, respectively, converts their profit functions to

$$u_1(a, b|\bar{c}_1) = u_1(a, b) + \left(\frac{-\beta}{2} + \frac{1}{4}\right)a = \beta\left(1 + \frac{b}{2}\right) - \frac{(a + 1)^2}{2} - \frac{(b - a)^2}{8} + \frac{a}{4} \quad (11)$$

and

$$u_2(a, b|\bar{c}_2) = u_2(a, b) + \left(\frac{\beta}{2} - \frac{1}{4}\right)b = \beta\left(1 - \frac{a}{2}\right) - \frac{(1 - b)^2}{2} - \frac{(b - a)^2}{8} - \frac{b}{4}. \quad (12)$$

Of course, since we have modified strictly concave functions by linear terms, the results continue to be strictly concave when $-1 \leq a < b \leq 1$. Next observe that

$$\frac{\partial u_1(a, b|\bar{c}_1)}{\partial a} = -(a+1) - \frac{a-b}{4} + \frac{1}{4} = -\frac{5a}{4} + \frac{b}{4} - \frac{3}{4} \tag{13}$$

and

$$\frac{\partial u_2(a, b|\bar{c}_2)}{\partial b} = 1 - b - \frac{b-a}{4} - \frac{1}{4} = \frac{a}{4} - \frac{5b}{4} + \frac{3}{4}; \tag{14}$$

as the system obtained by equating the right-hand sides of (13) and (14) has $(\bar{a}, \bar{b}) = (-\frac{1}{2}, \frac{1}{2})$ as its unique solution, we have that this is a Nash equilibrium solution. One can verify that no other point is a Nash equilibrium with respect to the modified problem. (But, the globally optimal point $(\frac{1}{2}, -\frac{1}{2})$ is a Nash equilibrium when considering the region $-1 \le b \le a \le 1$, under the utilities $u_2(-b, -a|\bar{c}_2)$ and $u_1(-b, -a|\bar{c}_1)$ for operators 1 and 2, respectively, where u_1 and u_2 are given by (6) and (7).)

3.1.5. Managerial Insights

The utilities of the two cart operators under the (unique) Nash equilibrium after the incentive scheme is implemented are equal to $\frac{5\beta}{4} - \frac{3}{8}$, compared with $\beta - \frac{1}{2}$ to both operators under the unique original Nash equilibrium at (0, 0). So, each gains $\frac{\beta}{4} + \frac{1}{8}$. The coefficients $\bar{c}_1$ and $\bar{c}_2$ correspond to bonuses that the retailers get for every foot they move away from the center. The source for these bonuses is the extra revenue gained from improved performance of the system.

3.2. Shortage Gaming

3.2.1. Description of the Problem

We consider a supply chain with n independent retailers over a single period. The retailers face random independent demands for a continuous product, and they have individual (constant) inventory costs for overage and underage. The retailers order the product from a single supplier who has a limited (fixed) amount. When the retailers' orders from the supplier exceed his stock, the latter applies a proportional allocation scheme to determine the quantities that each retailer would receive. Under this scheme, the quantities delivered to the retailers are proportional to their order sizes, respectively. The proportional allocation creates an incentive for the retailers to inflate their order sizes so as to tilt the allocation in their favor and get more of the product. This phenomenon is known in the supply chain literature as "shortage gaming" (see for example, [5, 7])—one of the contributors to the buildup of the bullwhip effect that is known to create havoc in the performance of supply chains.

We next introduce the formal parameters for the above model. First, without loss of generality, we assume that the supplier has a single unit of the product. Also, we denote the (positive) per-unit overage and underage cost parameters of retailer i by h_i and π_i, respectively. Next, the distribution of the demand that retailer i faces is assumed to have density $f_i(\cdot)$, and we let $F_i(\cdot)$ and μ_i denote the corresponding cumulative distribution function and the expectation. For simplicity, we assume that [0, 1] is the support of these density functions (this assumption avoids ties in the algebraic derivations; see the comment in the next subsection about its relaxation).

Finally, we assume that disposal of delivered amounts of the product is prohibited, that is, overage costs must be paid on all delivered items that are not sold. But, a similar analysis applies to the case where this assumption is relaxed.

3.2.2. The Retailers' Cost Function

We denote by q amounts that retailers order and by g amounts that are actually delivered to them. The cost that retailer i encounters when he receives $0 \le g_i \le 1$ units at the beginning of the period follows the familiar "newsboy formula,"

$$w_i(g_i) = h_i \int_0^{g_i} (g_i - x) f_i(x)\, dx + \pi_i \int_{g_i}^{1} (x - g_i) f_i(x)\, dx. \tag{15}$$

Evidently, $w_i'(g_i) = (h_i + \pi_i)F_i(g_i) - \pi_i$ and $w_i''(g_i = f_i(g_i)$ for $g_i \ge 0$. In particular, $w_i(0) = \pi_i\mu_i$, $w_i'(0) = -\pi_i < 0$, $w_i(1) = h_i(g - \mu_i)$, $w_i'(1) = h_i > 0$, w_i is continuously differentiable and strictly convex and it has a unique minimum at a point $0 < g_i^0 < 1$, which satisfies $w_i'(g_i^0) = 0$, that is, $F_i(g_i^0) = \frac{\pi_i}{h_i + \pi_i}$. The shape of w_i is illustrated in Figure 1. (All of the forthcoming analysis applies when the assumption that each retailer's demand is bounded by 1 is replaced by the assumption that the optimal stock levels (the g_i^0's) are uniquely defined and are below 1.) The decision variables for the retailers are the quantities they order. When the retailers order $q_1, \ldots, q_n$, respectively, the amount delivered to retailer i will be

$$g_i(q_1, \ldots, q_n) = \begin{cases} q_i & \text{if } \sum_{s=1}^{n} q_s \le 1 \\ \dfrac{q_i}{q_1 + \cdots + q_n} & \text{if } \sum_{s=1}^{n} q_s > 1, \end{cases} \tag{16}$$

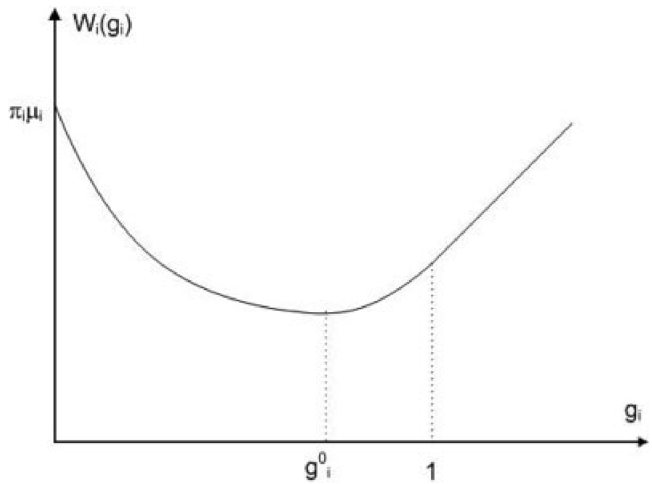

Figure 1. The cost function $w_i(g_i)$.

and the utility function u_i for retailer i is given by

$$u_i(q_1, \ldots, q_n) = -w_i[g_i(q_1, \ldots, q_n)]. \tag{17}$$

We observe that when $\sum_{s=1}^{n} q_s > 1$, the delivered quantities and the associated values $u_i(q_1, \ldots, q_n)$ are independent of scalar scaling.

Given order quantities $q_1, \ldots, q_n$ and a retailer i, we use the notation q_{-i} for $\sum_{s=1, s\neq i}^{n} q_s$. As $\sum_{s=1}^{n} q_s = q_i + q_{-i}$, we consider the functions g_i and u_i as functions of the pair (q_i, q_{-i}) and refer to $g_i(q_i, q_{-i})$ and $u_i(q_i, q_{-i}) = -w_i[g_i(q_i, q_{-i})]$, respectively.

We next consider each utility function $u_i(q_i, \bar{q}_{-i})$ as a function of q_i with $\bar{q}_{-i}$ fixed. It is shown in the Appendix that $u_i(\cdot, \bar{q}_{-i})$ has the following properties:

(i) $u_i(\cdot, \bar{q}_{-i})$ is continuous on $[0, \infty)$ and continuously differentiable on $[0, \infty)\backslash\{1 - \bar{q}_{-i}\}$;

(ii) $(u_i)'_+(1 - \bar{q}_{-i}, \bar{q}_{-i}) = [(u_i)'_-(1 - \bar{q}_{-i}, \bar{q}_{-i})]\bar{q}_{-i}$, where $(u_i)'_-$ and $(u_i)'_+$ denote, respectively, the left and right derivatives of $u_i(\cdot, \bar{q}_{-i})$;

(iii) $u_i(\cdot, \bar{q}_{-i})$ attains a unique global maximum, say at $\bar{q}_i^*$, which is its only local maximum. Further, with $\gamma_i \equiv \frac{\bar{q}_{-i} g_i^0}{1 - g_i^0}$, if $g_i^0 \leq 1 - \bar{q}_{-i}$ then $\bar{q}_i^* = g_i^0$, and if $g_i^0 > 1 - \bar{q}_{-i}$ then $\bar{q}_i^* = \gamma_i \geq g_i^0$. In both cases, $g_i^0 \leq \bar{q}_i^*$;

(iv) $u_i(\cdot, \bar{q}_{-i})$ is concave and increasing on $[0, \bar{q}_i^*]$ and is decreasing on $[\bar{q}_i^*, \infty)$.

(Of course, $\bar{q}_i^*$ depends on $\bar{q}_{-i}$ but, for convenience, we avoid writing $\bar{q}_i^*(\bar{q}_{-i})$.) The above properties of $u_i(\cdot, \bar{q}_{-i})$ are illustrated in Figure 2.

3.2.3. Globally Optimal and Nash Equilibrium Solutions

Let $(\bar{g}_1, \ldots, \bar{g}_n)$ be (the unique) minimizer of $w(g_1, \ldots, g_n) \equiv \sum_{i=1}^{n} w_i(g_i)$ over the (compact) region $\{(g_1, \ldots, g_n) : \sum_{i=1}^{n} g_i \leq 1 \text{ and } g_i \geq 0 \text{ for each } i\}$. We observe that $\bar{g}_i \leq g_i^0$ for each i, for otherwise $\bar{g}_i$ can be replaced by g_i^0, resulting in a decrease of $w_i(\cdot)$ and of $w(\cdot)$. Also, if $\sum_{i=1}^{n} g_i^0 \leq 1$, then $(\bar{g}_1, \ldots, \bar{g}_n) = (g_1^0, \ldots, g_n^0)$ and, in this case, $(\bar{q}_1, \ldots, \bar{q}_n) = (\bar{g}_1, \ldots, \bar{g}_n)$ is both globally optimal and a Nash equilibrium solution. This case

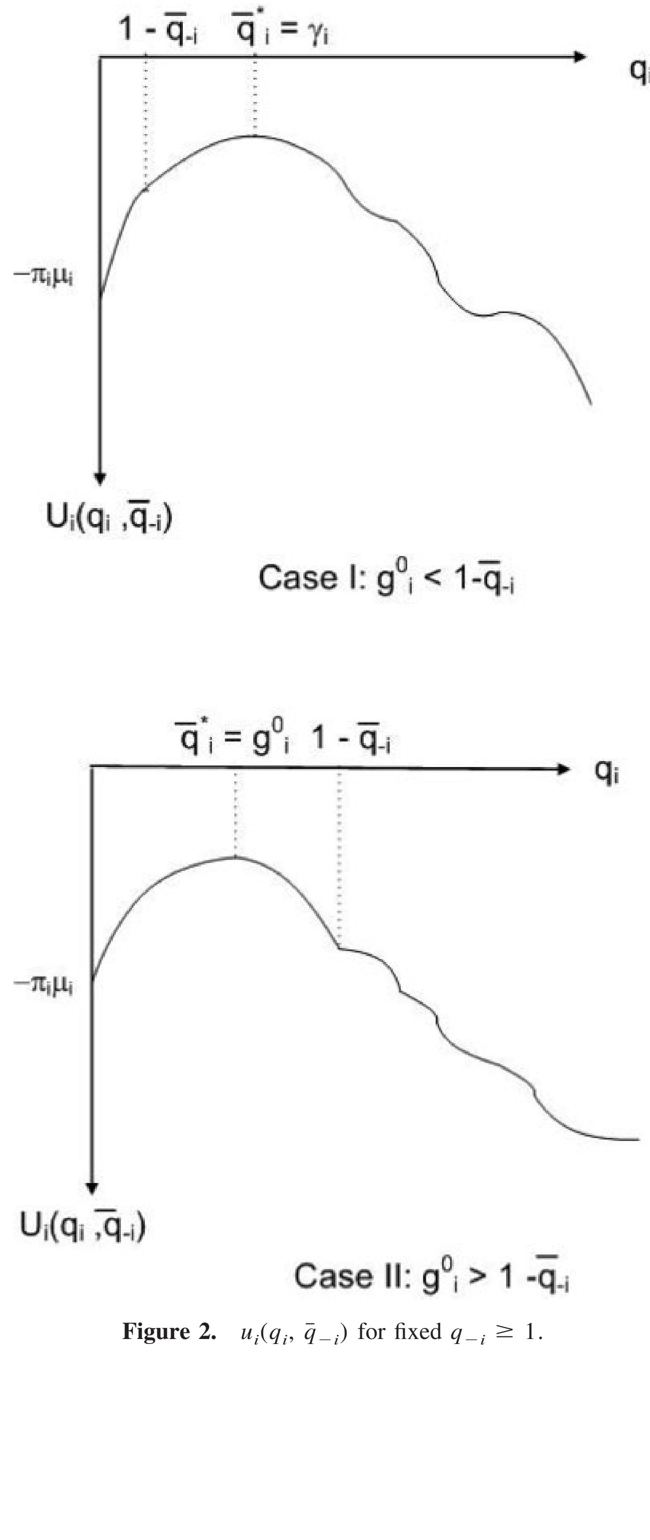

Figure 2. $u_i(q_i, \bar{q}_{-i})$ for fixed $q_{-i} \geq 1$.

is not interesting, and we will therefore restrict attention only to the case where $\sum_{i=1}^{n} g_i^0 > 1$. We conclude that

(i) $\bar{g}_i < g_i^0$ for at least one retailer (as $\sum_{i=1}^{n} \bar{g}_i \le 1 < \sum_{i=1}^{n} g_i^0$);
(ii) $\sum_{i=1}^{n} \bar{g}_i = 1$ (for if $\sum_{i=1}^{n} \bar{g}_i < 1$, a retailer i with $\bar{g}_i < g_i^0$ can improve $w_i(\cdot)$ and $w(\cdot)$ by increasing his share);
(iii) for each $\alpha \ge 1$, $(\bar{q}_1, \ldots, \bar{q}_n) \equiv \alpha(\bar{g}_1, \ldots, \bar{g}_n)$ maximizes the global utility function $u = \sum_{i=1}^{n} u_i$ (yielding allocations $\bar{g}_1, \ldots, \bar{g}_n$, respectively, to the retailers);
(iv) there is no Nash equilibrium. Consider a vector $(q_1, \ldots, q_n)$. As $\sum_{i=1}^{n} g_i(q_1, \ldots, q_n) \le 1 < \sum_{i=1}^{n} g_i^0$, there exists a retailer i with $g_i(q_1, \ldots, q_n) < g_i^0$; such a retailer has an incentive to change his order q_i to $\frac{q_{-i} g_i^0}{1 - g_i^0}$, increasing his allocation to g_i^0.

Note that the assumption $\sum_{i=1}^{n} g_i^0 > 1$ does not exclude the existence of a retailer i with $\bar{g}_i = g_i^0$; of course, such a retailer cannot improve his utility by a unilateral move and there is no need to introduce a reward mechanism to influence his incentives. Also, it is possible to have $\bar{g}_i = 0$.

3.2.4. *Implementation of the Linear Reward Scheme*

Consider the globally optimal solution $(\bar{q}_1, \ldots, \bar{q}_n) \equiv \alpha(\bar{g}_1, \ldots, \bar{g}_n)$ with $\alpha > 1$. Following (3) and (4), for each retailer i, we consider the linear reward coefficient $\bar{c}_i$ given by

$$\bar{c}_i = \sum_{\substack{t=1\\t\neq i}}^{n} \frac{\partial u_t}{\partial q_i}\bigg|_{(q_1,\ldots,q_n)=(\bar{q}_1,\ldots,\bar{q}_n)}. \qquad (18)$$

Using (16) and (17) and $\bar{q}_i + \bar{q}_{-i} = \alpha > 1$, we have an explicit representation of $\bar{c}_i$ given by

$$\bar{c}_i = \sum_{\substack{t=1\\t\neq i}}^{n} \left[w_t'\left(\frac{\bar{q}_t}{\bar{q}_i + \bar{q}_{-i}}\right)\right]\left[\frac{\bar{q}_t}{(\bar{q}_i + \bar{q}_{-i})^2}\right] = \sum_{\substack{t=1\\t\neq i}}^{n} [w_t'(\bar{g}_t)]\left(\frac{\bar{q}_t}{\alpha^2}\right)$$
$$= \sum_{\substack{t=1\\t\neq i}}^{n} [(h_t + \pi_t)F_t(\bar{g}_t) - \pi_t]\left(\frac{\bar{q}_t}{\alpha^2}\right). \qquad (19)$$

When $\bar{g}_i > 0$, $\bar{q}_i > 0$ and

$$0 = \frac{\partial u}{\partial q_i}\bigg|_{(q_1,\ldots,q_n)=(\bar{q}_1,\ldots,\bar{q}_n)} = \frac{\partial u_i}{\partial q_i}\bigg|_{(q_1,\ldots,q_n)=(\bar{q}_1,\ldots,\bar{q}_n)} + \sum_{\substack{t=1\\t\neq i}}^{n} \frac{\partial u_t}{\partial q_i}\bigg|_{(q_1,\ldots,q_n)=(\bar{q}_1,\ldots,\bar{q}_n)} = u_i'(\bar{q}_i, \bar{q}_{-i}) + \bar{c}_i. \qquad (20)$$

Further, as $\bar{q}_i = \alpha - \bar{q}_{-i} > 1 - \bar{q}_{-i}$ and $\bar{g}_i \le g_i^0$, we conclude from (20), from (16) and (17) and $\bar{q}_i + \bar{q}_{-i} = \alpha > 1$, from the convexity of w, and from $w_i'(g_i^0) = 0$ that

$$\bar{c}_i = -u_i'(\bar{q}_i, \bar{q}_{-i}) = \left[w_i'\left(\frac{\bar{q}_i}{\bar{q}_i + \bar{q}_{-i}}\right)\right]\left[\frac{\bar{q}_{-i}}{(\bar{q}_i + \bar{q}_{-i})^2}\right]$$
$$= [w_i'(\bar{g}_i)]\left(\frac{\bar{q}_{-i}}{\alpha^2}\right) \le [w_i'(g_i^0)]\left(\frac{\bar{q}_{-i}}{\alpha^2}\right) = 0. \qquad (21)$$

Implying that $[w_i'(\bar{g}_i)](\frac{\bar{q}_{-i}}{\alpha^2}) = [(h_i + \pi_i)F_i(\bar{g}_i) - \pi_i](\frac{\alpha - \bar{q}_i}{\alpha^2})$ is an explicit expression for $\bar{c}_i$ and that $\bar{c}_i \le 0$.

We shall verify in the Appendix that $(\bar{q}_1, \ldots, \bar{q}_n)$ is a unique Nash equilibrium with respect to the modified utility functions given by

$$u_i(q_1, \ldots, q_n|\bar{c}_i) = u_i(q_1, \ldots, q_n) + \bar{c}_i q_i \quad \text{for each retailer } i. \qquad (22)$$

We note that one cannot apply Theorem 1 of Rothblum [27] directly as the functions $u_i(\cdot)$ and $u_i(\cdot|\bar{c}_i)$ are not concave! Still, we examine each function $u_i(\cdot, \bar{q}_{-i}|\bar{c}_i)$ and prove that it attains a maximum at $\bar{q}_i$. Our proof relies on the following fact (established within the Appendix): for each $\bar{q}_{-i}$ and for each $\bar{c}_i \le 0$, $u_i(\cdot, \bar{q}_{-i}|\bar{c}_i)$ is quasi-concave over $[0, \bar{q}_i^*]$ (of course, $\bar{q}_i^*$ depends on $\bar{q}_{-i}$). It is noted that quasi-concavity of a function $u_i(\cdot, \bar{q}_{-i}|\bar{c}_i)$ with respect to *all* $\bar{c}_i$'s (not just $\bar{c}_i \le 0$) is equivalent to concavity.

3.2.5. *Managerial Insights*

The effect of the linear rewards in this case is to penalize the retailers for increasing their orders. These penalties effectively eliminate the motivation to state inflated orders. This effect is similar to the "truthfully implementable direct revelation" or "strategy proof" mechanisms in social choice theory (see, e.g., [20], Ch. 23), which are implemented in various settings such as bidding procedures (e.g., [36]); see also the ad hoc use of this approach in queuing systems (e.g., [21]). Still, one key difference between the approach proposed here and the mechanisms mentioned just above is that there is no information asymmetry in the settings we assume here. Thus, even though all agents have access to the complete set of information, no one can do better than order his true requirement. Finally, we note that linear reward/penalty schemes are in fact practiced in many bidding systems where there are fees that are proportional to the bid amount (for example, where bidders are required to deposit a certain proportion of the bid and incur an interest cost that is proportional to the bid they place).

3.3. Retailers' Coordination of Supplier Selection

3.3.1. *Description of the Problem*

We consider n identical retailers, each facing a deterministic demand of one unit per period of a continuous commodity, e.g., oil. The retailers can order from two available suppliers. The first supplier offers any quantity of the commodity at a fixed price of $\$(V + W)$ per unit. The second supplier offers a price that is an increasing function of the demand and equals $\$[V + W(\frac{x}{n})^\alpha]$ per unit where $x \in [0, n]$ is the total demand he is asked to supply and $\alpha \geq 1$ is a fixed parameter. Such pricing is common in production situations with limited capacities—the first batch of units is produced on the cheapest machine until its capacity is exhausted, the second batch is produced on the second cheapest machine, and so on. Here, we discuss an extreme case where the increase in the price per unit is linear. We denote by x_i the amount of demand that retailer i will purchase from the second supplier (hence, $(1 - x_i)$ is the amount purchased from the first supplier).

3.3.2. *The Retailer's Cost Function*

As each retailer policy is independent of V, we will assume, without loss of generality, $V = 0$ and $W = 1$. The utility function of retailer i is then given by

$$u_i(x_1, \ldots, x_n) = -(1 - x_i) - x_i\left(\frac{\sum_j x_j}{n}\right)^\alpha. \tag{23}$$

We then have

$$\frac{\partial u_i(x_1, \ldots, x_n)}{\partial x_i} = 1 - \frac{(\sum_j x_j)^\alpha}{n^\alpha} - \frac{x_i\alpha(\sum_j x_j)^{\alpha-1}}{n^\alpha} \tag{24}$$

and

$$\frac{\partial^2 u_i(x_1, \ldots, x_n)}{\partial (x_i)^2} = -\left[\frac{2\alpha(\sum_j x_j)^{\alpha-1}}{n^\alpha} + \frac{x_i\alpha(\alpha - 1)(\sum_j x_j)^{\alpha-2}}{n^\alpha}\right]. \tag{25}$$

In particular, each u_i is concave in x_i.

3.3.3. *Globally Optimal and Nash Equilibrium Solutions*

With $x \equiv \sum_j x_j$, the global utility function is given by

$$u(x_1, \ldots, x_n) = -\sum_i (1 - x_i) - \frac{(\sum_i x_i)^{\alpha+1}}{n^\alpha} = -n + x - \frac{x^{\alpha+1}}{n^\alpha}. \tag{26}$$

It follows that $(x_1, \ldots, x_n)$ is globally optimal if and only if $x = n(\frac{1}{1+\alpha})^{1/\alpha}$ and the unique symmetric globally optimal solution is

$$(\bar{x}_1, \ldots, \bar{x}_n) = [(1 + \alpha)^{-1/\alpha}](1, \ldots, 1); \tag{27}$$

in particular, for $\alpha = 1$, $\bar{x} = (\frac{1}{2}, \ldots, \frac{1}{2})$ is the unique globally optimal solution. We further observe that a necessary condition for a Nash equilibrium solution is that for each $i = 1, \ldots, n$, the right-hand side of (24) equals zero. With $x \equiv \sum_j x_j$, these conditions assure that $x^\alpha = \frac{n^{\alpha+1}}{n+\alpha}$ and $x_i = (\frac{n}{n+\alpha})^{1/\alpha}$ for each i; using concavity one can verify that $(\frac{n}{n+\alpha})^{1/\alpha}(1, \ldots, 1)$ is a unique Nash equilibrium solution.

3.3.4. *Implementation of the Linear Reward Scheme*

Consider the case $\alpha = 1$ and the symmetric globally optimal solution $(\bar{x}_1, \ldots, \bar{x}_n) = (\frac{1}{2}, \ldots, \frac{1}{2})$. To apply the linear reward scheme given by (3) and (4), use (23) to set

$$\bar{c}_i = \sum_{j \neq i} \left.\frac{\partial u_j}{\partial x_i}\right|_{(x_1, \ldots, x_n)=(1/2, \ldots, 1/2)} = -\sum_{j \neq i} \left.\frac{x_j}{n}\right|_{x_j = 1/2} = -\frac{(n-1)}{2n}. \tag{28}$$

The modified utility function of retailer i is then given by

$$u_i(x_1, \ldots, x_n | \bar{c}_i) = -1 + x_i - \frac{(x_i)^2}{n} - \frac{x_i(\sum_{j \neq i} x_j)}{n} - \frac{(n-1)}{2n} x_i; \tag{29}$$

a necessary condition for a Nash equilibrium solution is then

$$1 - \frac{2x_i}{n} - \frac{\sum_{j \neq i} x_j}{n} - \frac{n-1}{2n} = 0 \qquad \text{for } i = 1, \ldots, n. \tag{30}$$

Summing (30) over $i = 1, \ldots, n$ we see that $x = \sum_i x_i$ must satisfy $n - \frac{2x}{n} - \frac{(n-1)}{n}x - \frac{n-1}{2} = 0$, that is, $x = \frac{n}{2}$, and (30) implies that $1 - \frac{x_i}{n} - \frac{x}{n} - \frac{n-1}{2n} = 0$, that is

$$x_i = n\left[1 - \frac{x}{n} - \frac{n-1}{2n}\right] = n\left[2 - \frac{1}{2} - \frac{n-1}{2n}\right] = \frac{1}{2}. \quad (31)$$

It is now easy to verify that $(\bar{x}_1, \ldots, \bar{x}_n) = (\frac{1}{2}, \ldots, \frac{1}{2})$ is a unique Nash equilibrium solution under the modified utility functions.

A similar analysis shows that for $\alpha > 1$, (23) yields for each $i = 1, \ldots, n$

$$\bar{c}_i = (n-1)\frac{\bar{x}_i}{n^\alpha}\alpha\bar{x}^{\alpha-1} = \frac{\alpha(n-1)}{n^\alpha(1+\alpha)}, \quad (32)$$

and with this linear reward/penalty scheme, (27) yields a unique Nash equilibrium.

3.3.5. *Managerial Insights*

As we observed, without coordination each retailer hopes to obtain a favorable rate from the second supplier and the aggregate outcome is that too much is being ordered from the second supplier. This situation resembles a transportation problem with many travelers who must choose between going in a certain route that is not affected by the traffic load (i.e., fixed travel time) or going through a second route where the travel time increases with the traffic load until it reaches its worse value, which is equal to the fixed travel time of the first route. Without coordination, the travelers will flock to the second route, causing congestion and losing the possible gains from spreading the traffic load over the two routes. Our coordination scheme is therefore analogous, in a sense, to a toll that is imposed on any traveler who selects the second route. In this context, with travelers using randomized strategies, our analysis shows that the use of marginal cost analysis to determine toll values will convert a globally optimal distribution of traffic to be stable—that is, no traveler will have an incentive to change his route-selection decision.

3.4. Retailer–Supplier Price-Order Quantity Coordination

3.4.1. *Description of the Problem*

Here we address a standard relation between a retailer and a supplier; see Cachon [4], among others. The supplier must determine the wholesale per-unit price that he will charge the retailer. Given the wholesale price and the parameters of the random demand for a single product, the retailer must decide how much to order. The two can either operate independently or coordinate their moves with the goal of gaining better performance for the combined supply chain that contains both of them. This is a typical vertical system. The issue we explore concerns the coordination between the two entities in the system.

We next detail the operation of the system. The retailer faces demand $D(\geq 0)$ for the single product with (continuous) distribution function F and density function f. The retailer sells the products at per unit market-driven price \$$P$, which is not affected by the level of his sales. The retailer buys the items at a per-unit price \$$w$ from the supplier, who produces the items at per-unit cost \$$C$. When the supplier and retailer operate independently, we must have $C \leq w \leq P$ (for otherwise there would be no incentive for the supplier to produce and/or for the retailer to sell). We address a single period problem in which the sequence of events is as follows: the supplier announces w; the retailer determines the order quantity q; the supplier produces q units and sells them to the retailer; the retailer sells q units or less (depending on the demand-realization); leftover units have no salvage value.

3.4.2. *Reduction of the Model to Single Order-Quantity Selection by the Retailer*

The supplier's decision variable is the price w he charges the retailer, whereas the decision variable of the retailer is a function δ, which maps the supplier's selected price into an order quantity. The expected profits of the retailer and the supplier are determined by w and by the actual order quantity q of the retailer; they are given, respectively, by

$$u_r(q, w) = -wq + P\left\{[1 - F(q)]q + \int_0^q xf(x)\,dx\right\}$$
$$\text{and} \qquad u_s(q, w) = (w - c)q. \quad (33)$$

Considering the action space Δ of the retailer as the set of functions δ mapping w into q, we have that the utility functions of the retailer and supplier can be expressed as

$$v_r(\delta, w) = u_r[\delta(w), w] \qquad \text{and} \qquad v_s(\delta, w) = u_s[\delta(w), w]. \quad (34)$$

A function $\delta \in \Delta$ is called degenerate if it is invariant of the supplier's price selection. With $u = u_r + u_s$ and $v = v_r + v_s$, we make the following observations:

(i) if $(\bar{\delta}, \bar{w})$ maximizes v then $[\bar{\delta}(\bar{w}), \bar{w}]$ maximizes u;
(ii) if $(\bar{q}, \bar{w})$ maximizes u and $\tilde{\delta}$ is the degenerate function mapping each w into $\bar{q}$, then $(\tilde{\delta}, \bar{w})$ maximizes v;

(iii) if $(\bar{q}, \bar{w})$ is a Nash equilibrium with respect to functions u_r and u_s and $\tilde{\delta}$ is the degenerate function mapping each w into $\bar{q}$, then $(\tilde{\delta}, \bar{w})$ is a Nash equilibrium with respect to v_r and v_s. Indeed, for every $\delta \in \Delta$,

$$v_r(\delta, \bar{w}) = u_r[\delta(\bar{w}), \bar{w}] \leq u_r(\bar{q}, \bar{w}) = v_r(\tilde{\delta}, \bar{w}),$$

and for every w,

$$v_s(\tilde{\delta}, w) = u_s[\tilde{\delta}(w), w] = u_s(\bar{q}, w) \leq v_s(\tilde{\delta}, \bar{w}).$$

(In fact, given $(\bar{q}, \bar{w})$ and $\tilde{\delta}$ as above, we also have the inverse implication and if $(\tilde{\delta}, \bar{w})$ is a Nash equilibrium with respect to v_r and v_s, then $(\bar{q}, \bar{w})$ is a Nash equilibrium with respect to u_r and u_s.)

We note that observation (iii) holds when the utilities u_r and u_s are replaced by modifications that are determined by the linear schemes given by (3) or (5). It follows that in searching for Nash equilibrium solutions with respect to the modified utility functions it suffices to consider degenerate actions of the retailer and the corresponding utility functions u_r and u_s for the retailer and for the supplier. This means that if we show that any $(\bar{q}, \bar{w})$ that maximizes u becomes a Nash equilibrium with modified utility functions given by (3) and (4), then given any $(\bar{\delta}, \bar{w})$ that maximizes v, we have that $(\tilde{\delta}, \bar{w})$ with $\tilde{\delta}(w) = \bar{\delta}(w)$ for each w is a Nash equilibrium for the original model with corresponding modified utility functions that use $\bar{c}_r[\bar{\delta}(\bar{w}), \bar{w}]$ and $\bar{c}_s[\bar{\delta}(\bar{w}), \bar{w}]$, respectively. This reduction (and the above observations) applies whenever v_r and v_s are derived from functions u_r and u_s by (34) and it does not rely on the explicit form of u_r and u_s given in Eq. (33).

A Nash equilibrium for the problem where the retailer's action space is the function space Δ can be determined by the following process. Let $\delta^* \in \Delta$ be the function that associates with each w a maximizer over q of $u_r(q, w)$ and let w^* be a maximizer over w of $u_s[\delta^*(w), w]$; then (δ^*, w^*) is a Nash equilibrium. Indeed, for every $\delta \in \Delta$,

$$v_r(\delta, w^*) = u_r[\delta(w^*), w^*] \leq u_r[\delta^*(w^*), w^*] = v_r(\delta^*, w^*),$$

and for every w,

$$v_s(\delta^*, w) = u_s[\delta^*(w), w] \leq u_s[\delta(w), w] \leq u_s[\delta(w^*), w^*] = v_s(\delta^*, w^*).$$

This line of analysis is related to the principle-agent literature; e.g., see [15].

3.4.3. *The Retailer's Profit Function*

With the retailer's utility function u_r given by (33),

$$\frac{\partial u_r(q, w)}{\partial q} = -w + P(1 - F(q)) \qquad \text{and} \qquad \frac{\partial^2 u_r(q)}{\partial q^2} = -Pf(q) \leq 0. \quad (35)$$

It follows that u_r is concave in q and attains a maximum at a point q satisfying $F(q) = \frac{P-w}{P}$.

3.4.4. *The Supplier Profit Function*

With the supplier's utility function u_s given by (33), we trivially have that it is concave in w. Also, for a given positive q, u_s attains a maximum at $w = P$, and for $q = 0$, w can be selected arbitrarily.

3.4.5. *Globally Optimal and Nash Equilibrium Solutions*

The global profit function is given by

$$u(q, w) = u_r(q, w) + u_s(q, w) = -Cq + P\left\{[1 - F(q)]q + \int_0^q xf(x)\,dx\right\}. \quad (36)$$

As u is independent of w, and (as in (35)) $\frac{\partial^2 u_r(q)}{\partial q^2} = -Pf(q)$, we conclude that u is (jointly) concave in (q, w). Also, for (q, w) to be a globally optimal solution requires that $F(q) = \frac{P-C}{P}$ with no restrictions on w except for $C \leq w \leq P$. The optimal global profit is positive. Also, as $C \leq w$, we have that $\frac{P-C}{P} \geq \frac{P-w}{P}$ and therefore the globally optimal order quantity is greater than the order quantity, which is optimal for the retailer (equality holds only if the supplier sets $w = C$—something he would never do in an uncoordinated situation). Finally, note that all Nash equilibrium solutions for the original system have the form $(q, w = P)$ where $F(q) = 0$.

3.4.6. *The Implementation of the Linear Reward Scheme*

Consider any globally optimal solution $(\bar{q}, \bar{w})$. So, $F(\bar{q}) = \frac{P-\bar{w}}{P}$ and $C \leq \bar{w} \leq P$ is arbitrary. To set up the linear reward scheme given by (3) and (4), consider

$$\bar{c}_{\mathrm{r}} \equiv \left.\frac{\partial u_{\mathrm{s}}(q, w)}{\partial q}\right|_{(q,w)=(\bar{q},\bar{w})} = \left.\frac{\partial[(w-C)q]}{\partial q}\right|_{(q,w)=(\bar{q},\bar{w})} = \bar{w} - C \quad (37)$$

and

$$\bar{c}_{\mathrm{s}} \equiv \left.\frac{\partial u_{\mathrm{r}}(q, w)}{\partial w}\right|_{(q,w)=(\bar{q},\bar{w})} = -\bar{q}. \quad (38)$$

Using $\bar{c}_{\mathrm{r}}$ and $\bar{c}_{\mathrm{s}}$ as per-unit rewards for the actions taken by the retailer and the supplier, respectively, converts their profit functions to

$$\begin{aligned} u_{\mathrm{r}}(q, w|\bar{c}_{\mathrm{r}}) &= u_{\mathrm{r}}(q, w) + \bar{c}_{\mathrm{r}}q = -wq \\ &\quad + P\left\{[1 - F(q)]q + \int_0^q xf(x)\,dx\right\} + (\bar{w} - C)q \\ &= P\left\{[1 - F(q)]q + \int_0^q xf(x)\,dx\right\} - Cq, \quad (39) \end{aligned}$$

and

$$\begin{aligned} u_{\mathrm{s}}(q, w|\bar{c}_{\mathrm{s}}) &= u_{\mathrm{s}}(q, w) + \bar{c}_{\mathrm{s}}w = (w - C)q - \bar{q}w \\ &= -Cq + w(q - \bar{q}). \quad (40) \end{aligned}$$

As the modified supplier's profit is independent of his action w, $\bar{w}$ is optimal for the supplier with any potential action q of the retailer. In particular, $\bar{w}$ is optimal when the retailer selects $\bar{q}$. On the other hand,

$$\frac{\partial u_{\mathrm{r}}(q, w|\bar{c}_{\mathrm{r}})}{\partial q} = -C + P(1 - F(q)) \quad \text{and} \quad \frac{\partial^2 u_{\mathrm{r}}(q)}{\partial q^2} = -Pf(q) \le 0.$$

With $w = \bar{w}$, u_{r} attains a maximum at any point q satisfying $F(q) = \frac{P-C}{P}$. So, we have indeed that $(\bar{q}, \bar{w})$ is a Nash equilibrium solution of the supply chain system with the modified profit functions.

3.4.7. Managerial Insights

The proposed coordination scheme can be implemented only if the supplier will be compensated for charging the retailer the minimal possible wholesale price. Since the retailer's expected profit increases, a gain-sharing mechanism that will transfer some of the retailer's extra gain to the supplier is required. Cachon and Lariviere [7] give an example of such a cooperation between Blockbuster (a U.S.-based media retail chain) and its suppliers that led to increased revenues to both parties.

Such arrangements are not new in practice—in fact, they are commonly found in real-world supply contracts where the parties have agreed on the terms on the basis of intuition or past experience. The analysis provided above may be useful in leading the parties toward the global optimal solution where the "threat" of the penalties they might incur if they depart from it is sufficient to ensure that this solution stays stable.

4. SUMMARY

This paper addresses one of the main issues in supply chain management—how to induce better coordination among the various agents that play a role in the chain so as to improve the overall system's performance. The main contribution of the paper lies in the development of a general mechanism that induces optimal coordination through linear rewards and (or) penalties and the demonstration of its usefulness in the context of relations among agents in supply chains.

Future research may evolve in several directions. First, one can address the issue of profit sharing—how to allocate among the agents the extra profit gained when moving from decentralized behavior to optimal performance (recall (5) and the discussion therein). Second, we have assumed conditions of perfect information. In many situations, the agents' actions are difficult to observe or verify. Therefore, there is a need to develop coordination mechanisms for partial information settings. Third, we use linear prices to achieve stability and coordination, but the use of nonlinear rewards and penalties may allow one to obtain a uniform reward/penalty mechanism over nonsymmetric agents and capture responses to observed variables that represent partial information. Fourth, we considered only four specific scenarios. One might look at other scenarios described in the SCM literature (or other operations management areas) and implement the process demonstrated here. In fact, any formulation of a multiagent decision-making model in SCM (or other areas) may be suitable for the implementation of the linear reward/penalty schemes of (4). Fifth, one may focus on specific scenarios from the set we developed here and investigate them more in depth. For example, in the retailers' location example (Section 3.1) we did not specify alternative demand functions. One might further explore these examples by developing specific representations for various demand and distribution functions, suggest appropriate solution techniques, and demonstrate them through numerical experiments. Finally, in our review we mentioned that a number of SCM researchers have already pointed out the possibility of using linear rewards or penalties to induce coordination but that all of these suggestions lacked solid theoretical foundation. Now that we provided such a foundation, one might go back to the literature, identify all such earlier suggestions, classify them into groups or categories, and attempt to fit them into the framework proposed here.

Finally, we believe that our paper demonstrates, once again, the potential applications of game theoretical models and results in the area of SCM. Our framework relates to noncooperative games (hence, the Nash equilibrium approach). Other approaches (e.g., coalition formation models in cooperative games) might also lead to fruitful results when applied to SCM problems.

APPENDIX

In this Appendix, we complete the analysis of Example 3.3 (shortage gaming). We recall that the utility function u_i of retailer i is defined by Eqs. (16) and (17) and properties of w_i are listed following Eq. (15).

Properties of $u_i(\cdot, \bar{q}_{-i})$ with $\bar{q}_{-i}$ Fixed

We recall that $u_i(q_i, \bar{q}_{-i})$ is the utility of retailer i when he selects q_i and the other retailers' actions are fixed at $\{\bar{q}_s : s = 1, \ldots, n \text{ and } s \neq i\}$, respectively (here $\bar{q}_{-i} = \sum_{s=1, s\neq i}^{n} q_s$).

We start exploring $u_i(q_i, \bar{q}_{-i})$ under the assumption that $\bar{q}_{-i} < 1$. Now, using Eqs. (16) and (17), for $0 \leq q_i < 1 - \bar{q}_{-i}$,

$$\frac{\partial u_i}{\partial q_i}(q_i, \bar{q}_{-i}) = -[w_i'(q_i)] = -[(h_i + \pi_i)F_i(q_i) - \pi_i] \tag{41}$$

and

$$\frac{\partial^2 u_i}{\partial q_i^2}(q_i, \bar{q}_{-i}) = -w_i''(q_i) = -(h_i + \pi_i)f_i(q_i); \tag{42}$$

and for $q_i > 1 - q_{-i}$,

$$\frac{\partial u_i}{\partial q_i}(q_i, \bar{q}_{-i}) = -\left[w_i'\left(\frac{q_i}{q_i + \bar{q}_{-i}}\right)\right]\left[\frac{1}{q_i + \bar{q}_{-i}} - \frac{q_i}{(q_i + \bar{q}_{-i})^2}\right]$$
$$= -\left[w_i'\left(\frac{q_i}{q_i + \bar{q}_{-i}}\right)\right]\left[\frac{\bar{q}_{-i}}{(q_i + \bar{q}_{-i})^2}\right], \tag{43}$$

and

$$\frac{\partial^2 u_i}{\partial q_i^2}(q_i, \bar{q}_{-i}) = -\left[\left(w_i''\left(\frac{q_i}{q_i + q_{-i}}\right)\right)\right]\left[\frac{\bar{q}_{-i}}{(q_i + \bar{q}_{-i})^2}\right]^2$$
$$+\left[w_i'\left(\frac{q_i}{q_i + q_{-i}}\right)\right]\left[\frac{2\bar{q}_{-i}}{(q_i + q_{-i})^3}\right]. \tag{44}$$

With $(u_i)'_-$ and $(u_i)'_+$ denoting the left and right derivatives of $u_i(\cdot, \bar{q}_{-i})$, we have that

(I) $u_i(0, \bar{q}_{-i}) = -w_i(0) = -\pi_i\mu_i < 0$,
(II) $u_i(\cdot, \bar{q}_{-i})$ is continuous at $1 - \bar{q}_{-i}$ and $u_i(1 - \bar{q}_{-i}, \bar{q}_{-i}) = -w_i(\bar{q}_{-i}) < 0$,
(III) $\lim_{q_i\to\infty} u_i(q_i, \bar{q}_{-i}) = -w_i(1) < 0$,
(IV) $u_i'(0, \bar{q}_{-i}) = \pi_i > 0$,
(V) $u_1'(q_i, \bar{q}_{-i}) = -w_i'(q_i)$ for $0 < q_i < 1 - \bar{q}_{-i}$,
(VI) $u_i'(q_i, \bar{q}_{-i}) = -\left[w_i'\left(\frac{q_i}{q_i + \bar{q}_{-i}}\right)\right]\left[\frac{\bar{q}_{-i}}{(q_i + \bar{q}_{-i})^2}\right]$ for $q_i > 1 - \bar{q}_{-i}$,
(VII) $(u_i)'_-(1 - \bar{q}_{-i}, \bar{q}_{-i}) = -w_i'(1 - \bar{q}_{-i})$,
(VIII) $(u_i)'_+(1 - \bar{q}_{-i}, \bar{q}_{-i}) = -[w_i'(1 - \bar{q}_{-i})]\bar{q}_{-i}$,
(IX) $\lim_{q_i\to\infty} u_i'(q_i, \bar{q}_{-i}) = 0$.

Thus, $u_i(\cdot, \bar{q}_{-i})$ is continuous and it is continuously differentiable everywhere, except for possibly at $1 - \bar{q}_{-i}$, where the left and right derivatives of u_i are well defined and have the same sign. This proves properties (i) and (ii) listed in Subsection 3.3.

Let $\gamma_i \equiv \frac{q_{-i}g_i^0}{1 - g_i^0}$, that is, γ_i is the unique point where $g_i(\gamma_i, \bar{q}_{-i}) = g_i^0$. As

$$(1 - \bar{q}_{-i}) - \gamma_i = \frac{1 - \bar{q}_{-i} - g_i^0}{1 - g_i^0},$$

we have that $g_i^0 < / = / > 1 - \bar{q}_{-i}$ if and only if $\gamma_i < / = / > 1 - \bar{q}_{-i}$; moreover, when g_i^0 and γ_i are larger than $1 - \bar{q}_{-i}$, then necessarily $\gamma_i > g_i^0 > 1 - \bar{q}_{-i}$. We consider two cases:

(i) $g_i^0 \neq 1 - \bar{q}_{-i}$: In this case, observations (V)–(VIII) show that $u_i'(\cdot, \bar{q}_{-i})$ has exactly one sign switch that occurs at $g_i^0 \neq 1 - \bar{q}_{-i}$ if $g_i^0 < 1 - \bar{q}_{-i}$ and at γ_i if $g_i^0 > 1 - \bar{q}_{-i}$, and this sign switch is from positive to negative values; in particular, the left and right derivatives of u_i at $1 - \bar{q}_{-i}$ are nonzero, different, but have the same sign (see Figure 2).

(ii) $g_i^0 = 1 - \bar{q}_{-i}$: In this case, observations (V)–(VIII) show that $u_i'(q_i, \bar{q}_{-i}) > 0$ for $0 < q_i < 1 - \bar{q}_{-i}$, $u_i'(q_i, \bar{q}_{-i}) < 0$ for $q_i > 1 - \bar{q}_{-i}$ and the left and right derivatives of u_i at $1 - \bar{q}_{-i}$ are zero.

We conclude that $u_i(\cdot, \bar{q}_{-i})$ has a unique global maximum at g_i^0 if $g_i^0 \leq 1 - \bar{q}_{-i}$, and at γ_i if $g_i^0 > 1 - \bar{q}_{-i}$; further, $u_i(\cdot, \bar{q}_{-i})$ has no other local maximizer. So, if $\bar{q}_i^*$ is the (unique) maximizer of u_i, then $\bar{q}_i^* \geq g_i^0$. This completes the proof of property (iii) in Subsection 3.3.

We finally prove that u_i is concave on $[0, \bar{q}_i^*]$. Of course, Eq. (42) trivially assures that u_i is concave on $[0, 1 - \bar{q}_{-i}]$, implying the claim when $g_i^0 < 1 - \bar{q}_{-i}$. It remains to verify that the concavity of u_i extends to the interval $[1 - \bar{q}_{-i}, \gamma_i] = [1 - \bar{q}_{-i}, \bar{q}_i^*]$ when $1 - \bar{q}_{-i} < g_i^0$. Indeed, when $1 - \bar{q}_{-i} < g_i^0$, (VII), (VIII), and $w_i'(x) \leq 0$ for $0 \leq x \leq g_i^0$ imply that $(u_i)'_-(1 - \bar{q}_{-i}, \bar{q}_{-i})$ and $(u_i)'_+(1 - \bar{q}_{-i}, \bar{q}_{-i})$ are positive and $(u_i)'_-(1 - \bar{q}_{-i}, \bar{q}_{-i}) > (u_i)'_+(1 - \bar{q}_{-i}, \bar{q}_{-i})$. Further, for $1 - \bar{q}_{-i} < q_i < \gamma_i$ we have that $u_i'(q_i, \bar{q}_{-i}) > 0$, and Eqs. (43) and (44) and the convexity of w imply that

$$u_i''(q_i, \bar{q}_{-i}) = \frac{-(\bar{q}_{-i})}{(q_i + \bar{q}_{-i})^4}\left[w_i''\left(\frac{q_i}{q_i + \bar{q}_{-i}}\right)\right] + \frac{-2u_i'(q_i)}{q_i + \bar{q}_{-i}} < 0, \tag{45}$$

verifying that the concavity of u_i extends to the interval $[1 - \bar{q}_{-i}, \gamma_i]$. We note that u_i is not concave on $[\bar{q}_i^*, \infty)$; in fact, Eq. (44) implies that u_i is convex for sufficiently large q_i. This completes the proof of property (IV) listed in Subsection 3.3.

The analysis of $u_i(\cdot, \bar{q}_{-i})$ when $\bar{q}_{-i} \geq 0$ is simpler than the case where $\bar{q}_{-i} < 1$ as one can ignore the region $0 \leq q_i < 1 - \bar{q}_{-i}$. In particular, in this case, u_i attains a maximum at γ_i, which is its unique local maximizer, and u_i is concave on $[0, \gamma_i]$.

Verification that $(\bar{q}_1, \ldots, \bar{q}_n) = \alpha(\bar{g}_1, \ldots, \bar{g}_n)$ with $\alpha \geq 1$ is a Nash Equilibrium Solution with Respect to the Modified Utility Functions

We start by assuming that $\alpha > 1$ and $\bar{g}_i > 0$. Recall that $u_i(\cdot, \bar{q}_{-i})$ attains a maximum at a point q_i^* (where the dependence on $\bar{q}_{-i}$ is suppressed) and that $u_i(\cdot, \bar{q}_{-i})$ is concave and increasing on $[0, \bar{q}_i^*]$ and decreasing on $[\bar{q}_i^*, \infty)$. Of course, the concavity of $u_i(\cdot, \bar{q}_{-i})$ on $[0, \bar{q}_i^*]$ extends to $u_i(\cdot, \bar{q}_{-i}|\bar{c}_i)$. Also, as Eq. (21) shows that $u_i'(\bar{q}_i, \bar{q}_{-i}) \geq 0$, we have that $1 - \bar{q}_{-i} < \alpha - \bar{q}_{-i} = \bar{q}_i \leq \bar{q}_i^*$.

We conclude from Eq. (20), from the concavity of $u_i(\cdot, \bar{q}_{-i})$ on $[0, \bar{q}_i^*]$, from $u_i'(q_i, \bar{q}_{-i}) \leq 0$ for $q_i \geq \bar{q}_i^*$(as $u_i(\cdot, \bar{q}_{-i})$ is decreasing in this region) together with Eq. (21) that the behavior of $u_i'(\cdot, \bar{q}_{-i}|\bar{c}_i)$ is summarized by the following table, implying that $u_i(\cdot, \bar{q}_{-i}|\bar{c}_i)$ has a global minimum at $\bar{q}_i$.

Value of q_i	Value of $u_i'(\cdot, \bar{q}_{-i}|\bar{c}_i)$
$q_i = \bar{q}_i$	$u_i'(\bar{q}_i, \bar{q}_{-i}|\bar{c}_i) = u_i'(\bar{q}_i, \bar{q}_{-i}) + \bar{c}_i = 0$
$0 \leq q_i < \bar{q}_i$ with $q_i \neq 1 - \bar{q}_{-i}$	$u_i'(q_i, \bar{q}_{-i}|\bar{c}_i) \geq u_i'(\bar{q}_i, \bar{q}_{-i})|\bar{c}_i = 0$
$1 - \bar{q}_{-i}$ (assuming $1 \geq \bar{q}_{-i}$)	$(u_i)'_-(1 - \bar{q}_{-i}, \bar{q}_{-i}|\bar{c}_i) \geq (u_i)'_+(1 - \bar{q}_{-i}, \bar{q}_{-i}|\bar{c}_i)12w \geq u_i'(q_i, \bar{q}_{-i}|\bar{c}_i) = 0$
$\bar{q}_i < q_i \leq \bar{q}_i^*$	$u_i'(q_i, \bar{q}_{-i}|\bar{c}_i) \leq u_i'(\bar{q}_i, \bar{q}_{-i})|\bar{c}_i = 0$
$q_i \geq \bar{q}_i^*$	$u_i'(q_i, \bar{q}_{-i}|\bar{c}_i) = u_i'(q_i, \bar{q}_{-i}) + \bar{c}_i \leq 0 + 0 = 0$

We next consider the case with $\alpha > 0$ and $\bar{g}_i = 0$. In this case, $\bar{q}_i = 0$, Eq. (20) holds with the first equality replaced with the inequality "≥," Eq. (21) holds with the first equality replaced with the inequality "≤," and a summary of the behavior of $u_i'(\cdot, \bar{q}_{-i}|\bar{c}_i)$ is given by the following table, implying, again, that $u_i'(\cdot, \bar{q}_{-i}|\bar{c}_i)$ has a global maximum at $\bar{q}_i = 0$.

Value of q_i	Value of $u_i'(\cdot, \bar{q}_{-i}|\bar{c}_i)$
$q_i = 0$	$u_i'(0, \bar{q}_{-i}|\bar{c}_i) = u_i'(0, \bar{q}_{-i}) + \bar{c}_i \leq 0$
$0 < q_i \leq \bar{q}_i^*$	$u_i'(q_i, \bar{q}_{-i}|\bar{c}_i) \leq u_i'(0, \bar{q}_{-i}|\bar{c}_i) \leq 0$
$q_i \geq \bar{q}_i^*$	$u_i'(q_i, \bar{q}_{-i}|\bar{c}_i) = u_i'(q_i, \bar{q}_{-i}) + \bar{c}_i \leq 0 + 0 = 0$

This completes the proof that $(\bar{q}_1, \ldots, \bar{q}_n)$ is a Nash equilibrium with respect to the modified utility functions when $\alpha > 1$.

The treatment of the globally optimal solution $(\bar{q}_1, \ldots, \bar{q}_n) \equiv (\bar{g}_1, \ldots, \bar{g}_n)$ (the case with $\alpha = 1$) is more complex as the functions u_i are not differentiable at this point. Still, one can verify that this point is a Nash equilibrium solution with respect to the modified utility functions by using left and right derivatives to show that $u_i(\cdot, \bar{q}_{-i}|\bar{c}_i)$ is maximized at $\bar{q}_i$.

Uniqueness of Nash Equilibrium Solutions for the Modified Problem

Let $(\tilde{q}_1, \ldots, \tilde{q}_n)$ be a Nash equilibrium solution with respect to the modified utility functions $u_i(\cdot|\bar{c}_i)$. We recall the use of the notation $\tilde{q}_{-i}$ for $\sum_{s=1, s\neq i}^n \tilde{q}_s$. We recall from Subsection 3.3 that $\bar{g}_i \leq g_i^0 \leq \bar{q}_i^*$.

We next argue that $\sum_{i=1}^n \tilde{q}_i \geq 1$. Suppose that this is not the case and $\sum_{i=1}^n \tilde{q}_i < 1 = \sum_{i=1}^n \bar{g}_i$. In this case, $\tilde{g}_i = \tilde{q}_i$ for each retailer i and for at least one retailer $\tilde{g}_i = \tilde{q}_i < \bar{g}_i$. It then follows from the concavity of u_i on $[0, \bar{q}_i^*]$ and $\tilde{q}_i < \bar{g}_i \leq g_i^0 \leq \bar{q}_i^*$ that for sufficiently small $\varepsilon > 0$, $u_i(\tilde{q}_i + \varepsilon, \tilde{q}_{-i})$ is independent of $\tilde{q}_{-i}$ and $u_i(\tilde{q}_i + \varepsilon, \tilde{q}_{-i}) - u_i(\tilde{q}_i, \tilde{q}_{-i}) \geq [u_i'(\tilde{q}_i)]\varepsilon \geq [u_i'(\bar{q}_i)]\varepsilon = -\bar{c}_i\varepsilon$, implying that $u_i(\tilde{q}_i + \varepsilon, \tilde{q}_{-i}|\bar{c}_i) - u_i(\tilde{q}_i, \tilde{q}_{-i}|\bar{c}_i) \geq 0$, that is, retailer i can increase his modified utility by a unilateral move, contradicting the assertion that $(\tilde{q}_1, \ldots, \tilde{q}_n)$ is a Nash equilibrium solution.

We next argue that $\tilde{g}_i \equiv g_i(\tilde{q}_1, \ldots, \tilde{q}_n) \leq g_i^0$ for each i. Suppose $g_i^0 < \tilde{g}_i = \frac{\tilde{q}_i}{\tilde{q}_i + \tilde{q}_{-i}}$ and we will establish a contradiction. Indeed, a decrease of $\tilde{q}_i$ to $\frac{\tilde{q}_{-i}g_i^0}{1 - g_i^0}$ would result in a decrease retailer's i share from $\tilde{g}_i$ to g_i^0, an increase in u_i from $-w_i(\tilde{g}_i)$ to $-w_i(g_i^0)$, and an increase in the term $\bar{c}_i q_i$ (as $\bar{c}_i \leq 0$); as the total effect is an increase of $u_i(\cdot, \bar{q}_{-1}|\bar{c}_i)$, we get a contradiction to the assertion that $(\tilde{q}_1, \ldots, \tilde{q}_n)$ is a Nash equilibrium solution with respect to the modified utility functions.

Let $\beta \equiv \sum_{i=1}^n \tilde{q}_i \geq 1$ and $\bar{q} \equiv \beta\bar{g}$, and we will show that $\tilde{q}_i = \bar{q}_i$. Of course, we have $\sum_{i=1}^n \bar{q}_i = \beta\,(\sum_{i=1}^n \bar{g}_i) = \beta$. We first consider the case where $\beta > 1$. We consider two cases, the first of which has two subcases.

CASE I: $\tilde{q}_i > 0$. We observe that $\bar{q}_i = \beta\bar{g}_i < \beta g_i^0$ and $\tilde{q}_i = \beta\tilde{g}_i \leq \beta g_i^0$. Also, the optimality of the selection of $\tilde{q}_i$ implies that

$$0 = \left.\frac{\partial u_i(\cdot|\bar{c}_i)}{\partial q_i}\right|_{(q_1,\ldots,q_n)=(\tilde{q}_1,\ldots,\tilde{q}_n)} = \left.\frac{\partial u_i}{\partial q_i}\right|_{(q_1,\ldots,q_n)=(\tilde{q}_1,\ldots,\tilde{q}_n)} + \bar{c}_i. \quad (46)$$

Thus, if $\bar{q}_i > 0$, we get from Eq. (21), from Eq. (46), and from Eq. (43) that

$$\left[w_i'\left(\frac{\bar{q}_i}{\beta}\right)\right]\left(\frac{\beta - \bar{q}_i}{\beta^2}\right) = \left[w_i'\left(\frac{\bar{q}_i}{\bar{q}_i + \bar{q}_{-i}}\right)\right]\left[\frac{\bar{q}_{-i}}{(\bar{q}_i + \bar{q}_{-i})^2}\right] = \bar{c}_i$$
$$= \left.\frac{-\partial u_i}{\partial q_i}\right|_{(q_1,\ldots,q_n)=(\tilde{q}_1,\ldots,\tilde{q}_n)} = \left[w_i'\left(\frac{\tilde{q}_i}{\tilde{q}_i + \tilde{q}_{-i}}\right)\right]\left[\frac{\tilde{q}_{-i}}{(\tilde{q}_i + \tilde{q}_{-i})^2}\right]$$
$$= \left[w_i'\left(\frac{\tilde{q}_i}{\beta}\right)\right]\left(\frac{\beta - \tilde{q}_i}{\beta^2}\right). \quad (47)$$

Consider the function $v_i(\cdot)$ mapping $0 \leq q_i \leq \beta g_i^0$ into $v_i(q_i) = \left[w_i'\left(\frac{q_i}{\beta}\right)\right](\beta - q_i)$; in particular, $v_i'(q_i) = \left[w_i''\left(\frac{q_i}{\beta}\right)\right]\left(\frac{\beta - q_i}{\beta^2}\right) - w_i'\left(\frac{q_i}{\beta}\right)$ and Eq. (47) shows that $v_i(\bar{q}_i) = v_i(\tilde{q}_i)$. As w_i is strictly convex, we have that $w_i''\left(\frac{q_i}{\beta}\right) \geq 0$ and $w_i'\left(\frac{q_i}{\beta}\right) \leq w_i'(g_i^0) = 0$ for $0 \leq q_i \leq \beta g_i^0$. Thus, v_i' is strictly positive on $[0, \beta g_i^0)$. So, v_i is strictly increasing on $[0, \beta g_i^0]$ and Eq. (47) implies that $\tilde{q}_i = \bar{q}_i$.

A similar argument applies in the case where $\bar{q}_i = 0$, except that Eq. (47) is replaced by

$$[w_i'(0, \beta)]\left(\frac{\beta - 0}{\beta^2}\right) \geq \bar{c}_i = \left[w_i'\left(\frac{\tilde{q}_i}{\beta}\right)\right]\left(\frac{\beta - \tilde{q}_i}{\beta^2}\right),$$

and the strict monotonicity of v_i implies that $\bar{q}_i = 0 = \tilde{q}_i$.

CASE II: $\tilde{q}_i = 0$. The arguments of the second subcase of Case I, show that

$$\left[w_i'\left(\frac{\bar{q}_i}{\beta}\right)\right]\left(\frac{\beta - \bar{q}_i}{\beta^2}\right) = \bar{c}_i \leq \left[w_i'\left(\frac{0}{\beta}\right)\right]\left(\frac{\beta - 0}{\beta^2}\right),$$

and the monotonicity of v_i implies that $\tilde{q}_i = 0 = \bar{q}_i$.

The case where $\beta = 1$ follows the same arguments except that partial derivatives of u_i at $(\tilde{q}_1, \ldots, \tilde{q}_n)$ and $(\bar{q}_1, \ldots, \bar{q}_n)$ are replaced by right partial derivatives.

ACKNOWLEDGMENT

The authors express their thanks to Professor M. Haviv from The Hebrew University in Jerusalem for helpful comments on an earlier draft of this work.

REFERENCES

[1] Y. Bassok and R. Anupindi, Analysis of supply contracts with total minimum commitment, IIE Trans 29 (1991), 373–381.
[2] M. Beckmann, C.B. McGuire, and C.B. Winsten, Studies in the economics of transportation, Yale University Press, 1956.
[3] G.E. Bolton and A. Ockenfels, ERC: A theory of equity, reciprocity, and competition, Am Econ Rev 90 (2000), 166–193.
[4] G.P. Cachon, "Competitive supply chain inventory management," Quantitative models for supply chain management, S. Tayur, R. Ganesham, and M. Magazine (Editors), Kluwer, 1999.
[5] G.P. Cachon, "Supply chain coordination with contracts," The handbook of operations research and management science: Supply chain management, S. Graves and T. de Kok (Editors), Kluwer, 2003.
[6] G.P. Cachon and M.A. Lariviere, An equilibrium analysis of linear, proportional and uniform allocation of scarce capacity, IIE Trans 31 (1999), 835–849.
[7] G.P. Cachon and M.A. Lariviere, Turning the supply chain into a revenue chain, Harvard Bus Rev 79 (2001), 20–28.
[8] G.P. Cachon and S. Netessine, "Game theory in supply chain analysis," Handbook of quantitative supply chain analysis: Modeling in the E-business era, D. Simchi-Levi, S.D. Wu, and Z.J. Shen (Editors), Kluwer, 2004.
[9] G.P. Cachon and P.H. Zipkin, Competitive and cooperative inventory policies in a two stage supply chain, Manage Sci 45 (1999), 936–953.
[10] F. Chen, Z. Drezner, J.K. Ryan, and D. Simchi-Levi, "The bullwhip effect: Managerial insights on the impact of forecasting and information on the variability in a supply chain," Quantitative models for supply chain management, S. Tayur, R. Ganesham, and M. Magazine (Editors), Kluwer, 1999.
[11] S. Chopra and P. Meindl, Supply chain management: Strategy, planning, and operation, Prentice Hall, 2001.
[12] M. Christopher, Logistics and supply chain management: Strategies for reducing cost and improving service, Financial Times/Pitman, London, 1998.
[13] R. Cole, Y. Dodis, and T. Roughgarden, "Pricing network edges for heterogenous selfish users," Proceedings of the 35th annual ACM symposium on theory computing (STOC), 2003, pp. 521–530.
[14] G. Hardin, The tragedy of the commons, Science 162 (1968), 1243–1248.
[15] B. Holmström, Moral hazard and observability, Bell J Econ 10 (1979), 74–91.
[16] J.H. Kagel and A.E. Roth, The handbook of experimental economics, Princeton University Press, 1995.
[17] H.L. Lee, V. Padmanabhan, and S. Whang, Information distortion in a supply chain: The bullwhip effect, Manage Sci 43 (1997), 546–558.
[18] H.L. Lee and M.J. Rosenblatt, A generalized quantity discount pricing model to increase supplier's profits, Manage Sci 32 (1986), 1177–1185.
[19] E. Lindahl, Just taxation—A positive solution, de Gerechtigkeit der Besteuerung, Lund I (1919), 85–98.
[20] A. Mas-Colell, M.D. Whinston, and J.R. Green, Microeconomic theory, Oxford University Press, 1995.
[21] H. Mendelson and S. Whang, Optimal incentive-compatible priority pricing for the M/M/1 queue, Oper Res 38 (1990), 870–883.
[22] C.L. Munson, J. Hu, and M.J. Rosenblatt, Teaching the costs of uncoordinated supply chains, Interface 33 (2003), 24–39.
[23] S. Netessine and N. Rudi, Centralized and competitive inventory models with demand substitution, Oper Res 51 (2003), 329–335.
[24] A.C. Pigou, The economics of welfare, Macmillan, London, 1932.
[25] A. Rapoport and A. Chammah, Prisoner's dilemma, University of Michigan Press, 1965.
[26] G. Robins, Pushing the limits of VMI, Store 77 (1995), 42–44.
[27] U.G. Rothblum, Optimality vs. equilibrium: Inducing optimality by linear rewards and penalties (2004), unpublished.
[28] A. Rubinstein, Perfect equilibrium in a bargaining model, Econometrica 50 (1982), 97–110.
[29] P.A. Samuelson, The pure theory of public expenditure, Rev Econ Statist 36 (1954), 387–389.
[30] R.J. Shonberger, Strategic collaboration: Breaching the castle walls, Bus Horiz 39 (1996), 20.
[31] D. Simchi-Levi, P. Kaminsky, and E. Simchi-Levi, Designing and managing the supply chain, Irwin McGraw–Hill, 2000.
[32] J.F. Shapiro, Modeling the supply chain, Brooks/ColeThomson Learning, Pacific Grove, CA, 2001.
[33] T.A. Taylor, Supply chain coordination under channel rebates with sales effort effects, Manage Sci 48 (2002), 992–1007.
[34] S. Tayur, R. Ganesham, and M. Magazine, Quantitative models for supply chain management, Kluwer, 1999.
[35] H.R. Varian, A solution to the problem of externalities when agents are well-informed, Am Econ Rev 84 (1994), 1278–1293.
[36] W. Vickery, Counterspeculations, auctions, and competitive sealed tenders, J Finance 16 (1961), 15–27.