A GOAL PROGRAMMING INVENTORY CONTROL MODEL APPLIED AT A LARGE CHEMICAL PLANT

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Haifa Chemicals Ltd. (HCL) is one of Israel's largest chemical manufacturing plants. Its annual production volume reaches several hundred thousand tons, valued at more than US $130 million. HCL's production facilities are located in the Haifa Bay area while its warehouses are spread around the world.

In recent years the world market for HCL's products has witnessed an increase in competition. With improvement in means of transportation, increase in the availability of data, and increase in the number of producers, HCL recognized the need to improve its production and inventory planning in several potential problem areas:

- Although some products are interrelated in their production and transportation processes, there was no overall (and simultaneous) evaluation of the production/inventory issues concerning these products.
- Although decisions were coordinated by department managers (production, marketing, finance, etc.), there was no systematic trade-off analysis of the potential outcomes with respect to the departments' interests.
- Occasionally, production planners were "surprised" by some limitation (e.g., transportation schedule, capacity overflow, etc.) whose effect could have been prevented by systematic analysis undertaken a few months earlier.
- HCL management realized that it is facing several conflicting objectives which are very difficult to quantify into a unidimensional objective function:
    - Meeting ordinary demand with certainty and providing high service level to occasional demand. The former is generated by regular clients whose orders are known in advance (according to annual contracts) and is coordinated with the freighter schedule. Failure to meet demand (i.e., stockouts) brings about the loss of immediate sales and causes customer ill will.
    - Keeping inventories at the lowest possible levels (in particular, below the company's own storage area capacity) so as to minimize inventory costs.
    - Maintaining stable production schedules with similar work loads on the different production facilities.

A linear goal programming (GP) model was constructed to provide HCL with a reliable production/inventory managerial control tool. Although linear programming models were introduced to the chemical industry in the 1950s and have since been in intensive use, they are typically limited to specific areas of production. They also usually consider a single objective (such as minimum cost) and are seldom characterized by a dynamic decision process (e.g., using a rolling horizon mechanism). The model developed for HCL has multiple objectives and is a dynamic procedure.

THE PRODUCT

The model is illustrated with one of HCL's prime export products, product Q, characterized by a relatively large production cost. Consequently, a large inventory of Q has a much greater economic significance in proportion to the overall production volume. Also, its profit contribution per ton is relatively large, making it extremely undesirable to lose orders or clients. The manufacturing line for Q operates continuously (on a 24 hours-a-day basis) and stops only for planned maintenance or unexpected breakdowns. Three important raw materials are involved in this line.

Material P, which is a by-product of one of HCL's main manufacturing lines (also sold by the company as a separate product), is prominent in terms of quantities. There is a choice between two other raw materials: H_1 is supplied by a local manufacturer according to a preplanned schedule and H_2 is imported by HCL to complement H_1.

Hence, when optimizing the production and storage levels of Q, one must determine dedicated quantities of its raw materials, i.e., the portion of the general stock of these materials which will not be used for purposes other than Q's production. Product Q is exported through consignment warehouses abroad. A detailed shipment plan for the planning horizon (constructed according to given time schedules for freighters) represents the demand for Q in the model. As occasional demand is supplied directly from the HCL plant at Haifa, no (safety) stock is required at the consignment warehouses except the operable stock, i.e., the quantities to be delivered to clients before the next replenishment takes place. In cases of emergency, when regular demand cannot be met, HCL has the option of purchasing Q in the open market, incurring the resulting loss of profit. However, no purchase is planned to meet occasional demand which cannot be satisfied otherwise.

A careful examination of the production and distribution systems of HCL identified only the following sources of uncertainties associated with Q: unscheduled sales (mostly to occasional customers), short breakdowns in the production process, and prolonged interruptions in the production process. Hence, safety stock was required to prevent disruptions in the supply process of Q. Safety stock of Q in Israel was computed by the "mean plus k standard deviations" rule. The value of k was determined to be approximately 2 according to holding costs and shortage penalties (see the newspaper boy model) [3]. The mean was determined by the expectation of unscheduled demand. The standard deviations reflect uncertainties due to unscheduled demand as well as interruptions in production.

HCL produces, stores, and distributes a variety of Q products, each of which may be produced in several grades and packaged in different ways. Safety stocks should consist of sufficient quantities of each of these items and grades. The mean and variance of the monthly unscheduled sales of each type (or grade) were estimated from available data records. These estimates were applied to the determination of safety-stock levels for each Q product, but the model considers Q as a single item and leaves the detailed production planning to Production Management. The mathematical development of the safety stock is available upon request to the first author or the *P&IMJ* Editorial Office.

HCL contracts with their regular clients allow the latter to determine the final timing of withdrawals from the warehouses. Accordingly, the quantities demanded for month t should be available at the beginning of the month. Since marine transportation from Haifa to the target harbors lasts at least two weeks, these quantities should be on their way no later than the middle of month $t - 1$. The model follows HCL's periodic review policy in using monthly decision points. This means that the quantity required in month t should be produced during month $t - 2$ and shipped in month $t - 1$ to meet the deadline of readiness at the beginning of month t. Thus, in fact, a buffer stock of about four weeks is maintained at each location. This additional inventory can be easily reduced by using smaller (e.g., weekly) time buckets.

THE MODEL

The model presented here is a multiproduct, multiperiod goal programming model. The products are Q and its raw materials. The periods were chosen as months according to ordinary planning procedures at HCL. The rolling planning horizon is four months. The objective is to minimize a weighted sum of deviations from specified goals so as to keep the inventory system within desired bounds by appropriate penalties which reflect the relative importance of the goals (given within the constraint set). Some of the goals are absolute (e.g., keeping inventory balance) and are associated with constraints without any deviation. Other goals are given by a goal number (e.g., safety stock quantity) and deviation variables in their constraints.

DISCUSSION OF THE MODEL DEFINITION

(Note that all equations referred to below can be found in the Appendix)

Q's Inventory balance: Inventory balance requires that the quantity stored at the beginning of each month plus the quantity produced during the month and the quantity purchased during the month should balance the Q quantities shipped during the month and the inventory at the beginning of the next month (Ignoring a negligible quantity wasted during handling). The balance equation (2) is given in a deterministic form either by using the average occasional demand, ADEM(Q), or by using actual orders by occasional clients if they have already been processed. Clearly, by the beginning of the next month, the opening inventory, INV(Q, t + 1), will not necessarily have the value computed by the model. Rather, its value will depend on the realization of the occasional demand and production in month t. Constraint (9) limits the unsupplied Q quantity (the dummy variable) and (10), (19) are the storage capacity constraints on Q.

Safety Stock Requirement: Using a detailed schedule, constraint (14) requires that only the shipments planned for the first period of the month plus the safety stock should be ready at the beginning of the month. Deviations below the safety level increase the shortage risk and should be penalized accordingly. On the other hand, deviations above that level increase the probability of incurring inventory holding cost.

Product P Requirements: Constraint (15) imposes P's inventory balance (with possible deviations) and (11), (20) its storage capacity. Limited storage capacity exists for the P quantities dedicated for the Q production. Since alternative outlets for P cannot be easily found at short notice, a minimal quantity of Q has to be produced in order to prevent a blockage of P and interruptions in the main process associated with P, or alternatives (such as additional storage) have to be used. Also, if shortages of P are experienced, material can be bought externally, but in limited quantities, at higher prices and with advance notice. Finally, it is assumed that the demand for P quantities for other HCL products besides Q are known to certainty levels which are not lower than those of Q.

H_1 and H_2 Requirements: The model considers the (known) supply schedules of both H_1 and H_2. It interprets any shortage of H_1 in terms of additional H_2 to be ordered from the foreign supplier. Surplus in any of the two alternatives is allowed, but is penalized appropriately. H_1 inventory balance is given by (3) and its storage capacity by (7), (17). Similarly, H_2 inventory balance is given by (16) and its storage capacity by (8), (18). Q's production balance with H_1 and H_2 is defined in (4) and the desired production ratio, i.e. the proportions of Q produced from H_1 and H_2, is imposed in (5)–(6).

Q's Manufacturing Facility Requirements: Q's manufacturing facility has some technological limitations which determine minimum and maximum production rates (denoted earlier as LBP(Q) and UBP(Q)). Inequalities (12)–(13) define Q's production capacity.

Criterion Function: Some of the penalties are directly derived from real economic costs, e.g. holding cost, order cost, etc. However, other penalties reflect management preferences with regard to possible outcomes of the model. The criterion function (1) is therefore a minimization of the penalties, weighted across time by a discount factor.

THE IMPLEMENTATION

The model presented is solved with an ordinary LP software on an IBM-PC. Solution times are negligible since the problem size does not exceed 100 LP rows. This section describes the results generated for two planning periods. The first was revisited four months in 1987 to allow comparisons with actual realizations. The second, a four month period in 1988, shows an attempt to use the model for real-time decisions. In both examples, the 'first period of the month' was taken as the first half of the month and demand was approximated as half of the monthly regular demand.

Total cost for the two periods amounted to 16200 and 220 "dollars" for the first and second periods respectively. In spite of the fact that not all of these amounts were real dollar costs, the bottom-line profit improvement implication is that, with the help of this inventory model, significant costs can be saved at HCL.

The common data parameters for both cases, derived from real data, are:

$$SS(Q) = 650; \quad TRAN(P, Q) = 0.585;$$
$$TRAN(H_1, Q) = 0.543; \quad TRAN(H_2, Q) = 0.735$$
$$LBC(Q) = LBC(P) = LBC(H_1) = LBC(H_2) = 0;$$
$$UBC(Q) = 4600, \quad UBC(P) = 1100,$$
$$UBC(H_1) = 470, \; UBC(H_2) = 970 \; DIS = 0.007;$$
$$LBP(Q) = 3.5; \; UBP(Q) = 5.9; \; ADEM(Q) = 130;$$
$$AUT(Q) = 0.92 \quad PEN_1 = 210 \quad PEN_2 = 7$$
$$PEN_3 = 4 \; PEN_4 = 4 \; PEN_5 = 240 \; PEN_6 = 50$$
$$PEN_7 = 8 \; PEN_8 = 22 \; PEN_9 = 0.2 \; PEN_{10} = 4$$

The values for the technological parameters (e.g., average utilization factor, production rates, capacities, etc.) were estimated from data taken over two consecutive years, 1986–1988. The values for the penalty parameters were taken in some cases according to real economic costs, and in other cases, estimated by management in a subjective manner. Thus for example:

PEN_1—Shortages in P were not experienced in recent years. Hence, managers had to estimate the additional costs involved in purchasing and importing this material. Their estimates averaged $210 per ton.

PEN_2—$7 per ton of P per month was quoted as the price for renting space at a nearby warehouse. The values of PEN_7, PEN_8, PEN_{10} were determined in a similar way.

PEN_3—Since safety stock was determined according to optimal service level calculated by the holding and shortage costs, PEN_3 and PEN_4, which represent tangents to the point of minimal cost, had to be equal. Theoretically, these penalties should increase quadratically when deviating in both directions from the optimum. However, such a formulation would have complicated the model and made it computationally infeasible under the limitations of the computer resources and the available time for planning. Hence, goal programming was used to bypass the problem by defining a goal number (SS(Q)), and using subjectively high weights to force the results to stay near this value.

PEN_5—This value ($240 / ton) reflects the actual loss of profit contribution.

PEN_6—The additional cost to purchase H_2 (relative to H_1) computed from observations of recent supply contracts for these materials.

PEN_9—After all the other penalties were presented for managerial approval, managers were asked to determine this value relative to the others. Their subjective evaluation of the damages associated with unsupplied irregular demand led to the value used in the example.

First Planning Period

Deviations from specified goals occurred only in two dimensions: surpluses in P and deviations below the safety stock requirements for Q. The sequence of positive deviations above maximal P storage capacity may indicate that the demand for Q is seriously lagging behind the production capacity of P. This should alert the Marketing division to initiate an effort to increase sales. (The profitability of such action can be measured by comparing the cost savings associated with the special disposal of the extra quantity of P to the investment, or price discounts, involved in the marketing action). If marketing efforts are infeasible, alternative uses of P should be closely examined. The deviations below the safety stock requirement imply that any breakdown in the manufacturing facility of Q or delays in the supply of its raw materials are potential causes for violations of supply contracts with Q's regular clients. One possible measure is to send a slow-down notice to Marketing. Extreme cases may provide a justification for an analysis of possible increase in the production rate upper limit. Alternatively, a delay in planned shutdown could be considered.

Initial inventory (1350) was very low compared to the requirement to hold inventory equivalent to the safety stock (650) plus half of the first month deliveries to regular clients (1750). This caused the model to gradually build up its Q inventories to the final level of 2450 tons. Inventory build-up was made possible only by leaving irregular demand unsupplied for the first three months of this period. However, no purchases of Q were required.

H_1 and H_2 supply schedules were adjusted as follows: H_1 supply was estimated at a level of 1300 tons per month in spite of the order of over 2100 tons per month. This adjustment was in light of a nation-wide shortage in H_1 which caused a proportional drop in deliveries to all customers. H_2 supply schedule was 800 tons per month except for the second month (1600 tons). There was no need to order additional quantities of H_2.

The final balance equation for the period is:

14890	+	1350	+	0
expected production in 4 months		initial inventory		purchase

= 16240 =	2450	+	14500	−	710
	expected final inventory		expected total demand in 4 months		expected unsupplied irregular demand

compared with the actual realization in this period:

14640	+	1350	+	0	= 15990
total production in 4 months		initial inventory		purchase	

≈ 15830 =	2430	+	13400	−	0
	final inventory		total demand in 4 months		total unsupplied irregular demand

Second Planning Period

No deviation is anticipated from the specified goals in any direction. Initially, there is a small surplus (25 tons) in Q inventory (caused by the given Q opening inventory), but it is smoothed in the next months. No problem is anticipated with the planned supply of P. H_1 supplies were estimated at a level of 1700 tons per month. As for H_2, an order for 800 tons was already placed for the first month and no other supplies of H_2 were planned in advance. Again, no additional quantities of H_2 were recommended for the remainder of this period.

TABLE 1: Condensed Sample Output, First Planning Period

Product Q

	Given Data			*Planned*				*Actual*		
Month	*Max Prod. Capab.*	*Total Sched. Demand*	*Irreg. Demand Out of Total*	*Opening Inventory*	*Prod.*	*Purchase*	*Unsuppl. Irreg. Demand*	*Opening Inventory*	*Shipment*	*Prod.*
1	3800	3750	250	1350	3779		250	1350	2000	3532
2	3600	3550	230	1629	3600		230	3670	5000	3331
3	3600	3550	230	1909	3600		230	1200	2700	3884
4	4000	3650	330	2189	3911		0	1980	3700	3892
5				2450				2430		
TOTAL		14500	1040		14890	0	710		13400	14639

Raw Materials P, H1 & H2

	P				*H1*		*H2*		
Month	*Opening Inventory*	*Sched. Supply*	*Shortage*	*Surplus*	*Opening Inventory*	*Sched. Supply*	*Opening Inventory*	*Sched. Supply*	*Planned Supply*
1	900	2500		0	161	1300	0	800	0
2	1189	2100		89	0	1300	0	1600	0
3	1183	2400		83	0	1300	714	800	0
4	1477	2100		377	463	1300	0	800	0
5	1289				0		312		
TOTAL		9100	0	549		5200		4000	0

The final balance equation for the period is:

13900	+	2500	+	0		
expected total produced in 4 months		initial inventory		purchase		
= 16400 =		2200	+	14200	−	0
		expected final inventory		expected total demand in 4 months		expected unsupplied irregular demand

Evaluation

In each example, the results reported above reflect only the analysis of the first month. The outcome of this analysis is decisions (production quantities, purchasing, etc.) for all 4 months in the planning horizon. However, only the immediate results are implemented. By immediate we mean those decisions whose deadline has arrived. These may include near-future decisions, (next month production quota) and medium-term decisions, (purchase order for the third month where the supplier requires a two-months notice). Most of the decisions for months 2–4 are reevaluated when the model is run again and updated decisions taken. In particular, the results of the second (real-time) example carried immediate managerial implications and served as indicators for the application of various controls at the discretion of the inventory system management. Managerial intervention and guidance were applied at critical decision points during the implementation phase and several decisions were changed a number of times.

Although it was not demonstrated by the above example one should be aware that the model may fail to yield a feasible solution in certain cases. In spite of the fact that the model is formulated as a goal programming model where most of the constraints have the flexibility to allow deviations, there might be instances where there are drastic gaps between planning (e.g. in terms of production hours, or scheduled shipments) and capacities. If this occurs, all planning parameters should be reevaluated and the program run again.

CONCLUSIONS

Production and inventory planning at a strategic level are often hampered by the inability of most

TABLE 2: Condensed Sample Output, Second Planning Period

	Product Q						
	Given Data			*Planned*			
Month	*Max Prod. Capab.*	*Total Sched. Demand*	*Irreg. Demand Out of Total*	*Opening Inventory*	*Prod.*	*Purchase*	*Unsuppl. Irreg. Demand*
1	4000	3950	300	2500	3620	0	0
2	4000	3350	250	2170	3390	0	0
3	3700	3450	270	2210	3450	0	0
4	3800	3450	270	2210	3440	0	0
5				2200			
TOTAL		14200	1090		13900	0	

	P				*H1*		*H2*		
Month	*Opening Inventory*	*Sched. Supply*	*Shortage*	*Surplus*	*Opening Inventory*	*Sched. Supply*	*Opening Inventory*	*Sched. Supply*	*Planned Supply*
1	640	2400	0	0	247	1700	0	800	0
2	922	2100	0	0	140	1700	584	0	0
3	1039	1900	0	0	0	1700	584	0	0
4	920	1690	0	0	0	1700	349	0	0
5	598				90		0		
TOTAL		8090	0			6800		800	0

managements to account for several (sometimes conflicting) objectives in a quantitative manner. The model that was constructed for HCL offers a way of handling some of these goals in a comprehensive manner and has the following advantages:

- The model analyzes several products and several periods simultaneously, thus considering mutual effects among products or periods.
- The model accounts for the conflicting objectives of the different departments at HCL. Marketing managers wish to increase inventories in order to cover all demand instances. Production managers wish to smooth production rates as much as possible. Finance managers wish to reduce inventories to save on holding costs and to schedule production according to cash flow. All these objectives (and more) are accommodated.
- The model offers a dynamic analysis across time, generating warning ahead of time. The repeated analysis of the same time period in a "window analysis" fashion, each time with more up-to-date information, allows tight control on performance and serves as a feedback tool to management.
- The model is characterized by considerable flexibility, allowing deviations in capacity, production, and safety stock limitations. What-if analysis is possible in a variety of ways, e.g., by changing the relative weights of the deviations.

To remain at the strategic level, the model refrains from distinguishing between the different grades of product Q in both demand and supply. A breakdown by grades of the production and shipment schedules is required at the tactical level in the implementation phase. The penalties used by the model to perform the tradeoffs among the goals were mostly economic-based. However, qualitative considerations which may be present at this planning level are expressed in terms of different sets of (sometimes subjective) penalties. The model given here was constructed only for one important product and its associated raw materials, but the same concepts can be used to extend the formulation so as to encompass all the materials involved in HCL's production. Since its installment at HCL, the model has helped management both in financial terms (cost reduction was realized mainly in smaller orders of expensive raw materials and smaller inventory holding costs) and in less tangible areas (e.g., more systematic procedures, better cooperation between departments, and better monitoring of managerial decisions).

EDITOR'S NOTE: I am in receipt of a letter from Mr. D. Sima, Vice President of Operations of the Haifa Chemicals Company. The letter verifies the implementation and cost reduction effects of the article. Actual dollar amounts were considerable but are unpublished due to reasons concerning competitive advantage.

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APPENDIX: MODEL, PARAMETERS, AND FORMULATION

The notation below is divided into three categories: data, policy parameters, and decision variables. The data items are needed at the beginning of each month for the monthly analysis. The policy parameters should be updated only once a year, or when particular events require a change.

INV(Q, 1): Product Q opening (on hand) inventory at the beginning of month 1.

INV(P, 1), INV(H_1, 1), INV(H_2, 1): Similarly, for dedicated quantities of materials P, H_1 and H_2 respectively.

SUP(P, t): Product P dedicated supply schedule in month t (t = 1, . . . , 4).

SUP(H_1, t), SUP(H_2, t): Similarly, for dedicated supplies of materials H_1 and H_2 respectively.

DEL(Q, t): Product Q's delivery schedule, i.e. quantities planned for shipping to regular clients in month t.

FDEL(Q, t): Product Q's deliveries planned for shipping to regular clients during the first period of month t.

PHR(Q, t): Planned production hours for product Q's line in month t.

Parameters

SS(Q)—Product Q safety stock level.

ADEM(Q)—Average monthly occasional demand.

AUT(Q)—mean (on-stream) utilization rate (yearly averaged).

UBP(Q), LBP(Q)—upper and lower bounds on Q's production rate capacity in (ton/hr)

UBC(i), LBC(i)—upper and lower bounds on storage capacities for material i, where i = Q, P, H_1, H_2.

TRAN(i, Q)—transformation factor of raw material i to Q (i.e. TRAN(i, Q) tons of material i are needed to produce one ton of Q).

MNRAT—minimal proportion of Q produced with H_1: (Q produced from H_1/(Q produced from H_2 + Q produced from H_1)).

MXRAT—maximal proportion of Q produced with H_1 (defined as above).

n—number of months in the planning horizon. (Here n = 4).

PEN_1—penalty per ton P to obtain it from external sources.

PEN_2—penalty per ton P to "dispose" of surplus beyond storage capacity.

PEN_3—penalty per ton Q deviating below safety stock quantity.

PEN_4—penalty per ton Q deviating above safety stock quantity.

PEN_5—penalty per ton to purchase Q abroad (when it is required to supply regular clients).

PEN_6—penalty per ton to purchase H_2.

PEN_7—penalty per ton H_1 beyond storage capacity.

PEN_8—penalty per ton H_2 beyond storage capacity.

PEN_9—penalty per ton of unsupplied occasional demand.

PEN_{10}—penalty per ton Q beyond storage capacity.

DIS—discount factor reflecting HCL financing costs (per month).

Variables

INV(Q, t)—Q's inventory at the beginning of month t (in HCL Israel locations).

INV(P, t), INV(H_1, t), INV(H_2, t)—similarly, for materials P, H_1, H_2 respectively.

PROD(Q, t)—Q's quantity (in tons) to be produced in month t.

$CDEV^+$(Q, t)—deviation above maximal Q storage capacity.

PUR(Q, t)—Q's quantity to be purchased abroad in month t.

DUM(Q, t)—dummy variable absorbing (without purchase) occasional demand in month t which cannot be supplied by Q's production.

$SDEV^+(Q, t)$—deviation above safety stock level at the beginning of month t.

$SDEV^-(Q, t)$—deviation below safety stock level at the beginning of month t.

$CDEV^-(P, t)$—deviation below minimal P storage capacity. When the quantity of P stored at the beginning of month t plus the quantity delivered through month t is insufficient for Q's production needs, $CDEV^-(P, t)$ represents P's quantity that HCL needs to obtain from external sources during month t.

$CDEV^+(P, t)$—deviation above maximal P storage capacity. When INV(P, t) is already at upper limit and extra P still exists, $CDEV^+(P, t)$ represents P's quantity that needs to be disposed of.

$QUAN(Q, H_1, t)$—Q's quantity produced with H_1 during month t.

$QUAN(Q, H_2, t)$—Q's quantity produced by H_2 during month t.

$CDEV^-(H_2, t)$—deviation below minimal H_2 storage capacity. When the quantities of H_1 and H_2 stored at the beginning of month t plus the quantities scheduled for delivery during month t are insufficient for Q's production needs, $CDEV^-(H_2, t)$ expresses the quantity (in terms of H_2) that HCL needs to obtain from external sources during month t.

$CDEV^+(H_1, t)$—deviation above maximal H_1 storage capacity. When $INV(H_1, t)$ is already at upper limit, and extra H_1 still exists, $CDEV^+(H_1, t)$ represents the H_1 quantity that requires renting additional storage.

$CDEV^+(H_2, t)$—deviation above maximal H_2 storage capacity (as above).

Mathematical Formulation

The constraints in the formulation are separated into ordinary ("physical") restrictions and "goal" constraints.

(1) $\text{Min} \sum_{t=1}^{n} (1 + DIS)^{-t} \cdot [PEN_1 \cdot CDEV^-(P, t) + PEN_2 \cdot CDEV^+(P, t) + PEN_3 \cdot SDEV^-(Q, t) + PEN_4 \cdot SDEV^+(Q, t) + PEN_5 \cdot PUR(Q, t) + PEN_6 \cdot CDEV^-(H_2, t) + PEN_7 \cdot CDEV^+(H_1, t) + PEN_8 \cdot CDEV^+(H_2, t) + PEN_9 \cdot DUM(Q, t) + PEN_{10} \cdot CDEV^+(Q, t))]$,

s.t.

physical restrictions

(2) $INV(Q, t) - INV(Q, t + 1) + PROD(Q, t) + PUR(Q, t) + DUM(Q, t) = DEL(Q, t) + ADEM(Q), t = 1, \ldots, n$

(3) $INV(H_1, t + 1) - INV(H_1, t) + TRAN(H_1, Q) \cdot QUAN(Q, H_1, t) = SUP(H_1, t), t = 1, \ldots, n$

(4) $QUAN(Q, H_1, t) + QUAN(Q, H_2, t) - PROD(Q, t) = 0, t = 1, \ldots, n$

(5) $(MNRAT - 1) \cdot QUAN(Q, H_1, t) + MNRAT \cdot QUAN(Q, H_2, t) \leq 0, t = 1, \ldots, n$

(6) $(1 - MXRAT) \cdot QUAN(Q, H_1, t) - MXRAT \cdot QUAN(Q, H_2, t) \leq 0, t = 1, \ldots, n$

(7) $-INV(H_1, t) \leq - LBC(H_1), t = 1, \ldots, n$

(8) $-INV(H_2, t) \leq - LBC(H_2), t = 1, \ldots, n$

(9) $DUM(Q, t) \leq ADEM(Q), t = 1, \ldots, n$

(10) $-INV(Q, t) \leq - LBC(Q), t = 1, \ldots, n$

(11) $-INV(P, t) \leq - LBC(P), t = 1, \ldots, n$

(12) $PROD(Q, t) \leq UBP(Q) \cdot PHR(Q, t) \cdot AUT(Q), t = 1, \ldots, n$

(13) $-PROD(Q, t) \leq - LBP(Q) \cdot PHR(Q,T) \cdot AUT(Q), t = 1, \ldots, n$

goal constraints

(14) $INV(Q, t) + SDEV^-(Q, t) - SDEV^+(Q, t) = SS(Q) + FDEL(Q, t), t = 1, \ldots, n$

(15) $INV(P, t + 1) - INV(P, t) - CDEV^-(P, t) + TRAN(P, Q) \cdot PROD(Q, t) = SUP(P, t), t = 1, \ldots, n$

(16) $INV(H_2, t + 1) - INV(H_2, t) - CDEV^-(H_2, t) + TRAN(H_1, Q) \cdot QUAN(Q, H_2, t) = SUP(H_2, t), t = 1, \ldots, n$

(17) $INV(H_1, t) - CDEV^+(H_1, t) \leq UBC(H_1), t = 1, \ldots, n$

(18) $INV(H_2, t) - CDEV^+(H_2, t) \leq UBC(H_2), t = 1, \ldots, n$

(19) $INV(Q, t) - CDEV^+(Q, t) \leq UBC(Q), t = 1, \ldots, n$

(20) $INV(P, t) - CDEV^+(P, t) \leq UBC(P), t = 1, \ldots, n$

non-negativity

(21) $CDEV^+(i, t), CDEV^-(i, t), INV(i, t), PROD(Q, t), PUR(Q, t), DUM(Q, t), SDEV^+(Q, t), QUAN(Q, H_1, t), QUAN(Q, H_2, t), SDEV^-(Q, t) \geq 0, i = Q, P, H_2, H_1, t = 1, \ldots, n$

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