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The Economic and Social Performance of Nations: Efficiency and Returns to Scale

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Abstract—The authors, who earlier have applied data envelopment analysis to rank the economic performance of nations, propose to extend the calculations to take account of social variables such as education, health, and welfare policy as well. An empirical application is presented, rating 72 developed and developing countries by their economic and social performance during the time period 1970 to 1985. For each country, the efficiency rating and also a measure of returns scale—increasing, constant, or decreasing returns to scale (RTS) are calculated. The frequency of alternate optima, leaving the returns of scale indeterminate, is examined. For nations with increasing RTS, continued long-run growth of both GNP and social performance is indicated. For countries with decreasing RTS, a slowing down of both GNP and social performance is indicated. The data for countries exhibiting constant RTS is further investigated using more detailed RTS analysis tools and, for some of these countries, we show how the constant RTS characterization can be highly sensitive to changes in the data. © 1997 Elsevier Science Ltd

MOTIVATION AND ANTECEDENTS

Newspapers sometimes rank nations in terms of their standard of living, Gross National Product (GNP), productivity, or other indicators of economic performance (e.g. Ref. [1]). The Council for Competitiveness in Washington D.C. publishes an annual assessment of the economic competitiveness of the group of advanced industrialized nations known as the G-7 nations. Employing a mathematical ranking technique called Data Envelopment Analysis, we recently used Council data to rank the performance of the G-7 nations during the time span 1972–1992 [2].

In the pages to follow, we turn our attention to the task of ranking the performance of a much larger set of nations—72 in all, including also developing and poor nations from all continents. Furthermore, we extend the criteria of ranking to include performance in terms of health, education, and social welfare. A comparison that focuses on strictly economic or financial variables might easily be viewed as skewed and stacked in favor of industrialized nations. Depending upon the political hue of government, wider social goals may, in many countries, actually overshadow the strictly commercial goals of national economic policy. In rating the overall performance of nations, then, we needed to identify a set of performance measures that imposes no *a priori* value judgment but incorporates a wide spectrum of possible goals of national policy.

Data Envelopment Analysis (DEA) was developed by Charnes, Cooper and Rhodes [3] as a methodology to evaluate the relative efficiency of Decision Making Units (DMUs). The DEA solution procedure involves the identification of a comparison point on the efficiency frontier that serves to evaluate the efficiency of the unit being analyzed ($=DMU_0$). The position of the frontier point is such that its outputs are greater than, or equal to, the corresponding outputs for DMU_0, while its inputs are smaller than, or equal to, those of DMU_0. The concept of efficiency as developed and calculated in this manner rests on well known concepts in microeconomic theory, including the notion of a feasible production set. Each feasible production point is written as a weighted average of observed DMUs. That is, the production possibility set and the efficiency frontier are constructed empirically from the observed DMUs.

Several recent studies have used DEA to rate the competitiveness and productive performance

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of entire nations. Using chance-constrained DEA, Land, Lovell and Thore [4] ranked 17 market-oriented West European economies and seven centrally planned East European economies. In other studies, Lovel and Pastor [5] ranked 16 Ibero-American countries while Lovell [6] evaluated 10 Asian countries.

As long as these rankings are limited to a comparison of GNP and its determinants, the mathematical envelopment procedure in effect amounts to estimating a piecewise linear production function of conventional form. But, when, as below, nations are ranked on the basis of a broad array of social performance indicators including mortality rates and school enrollment in secondary education, the theoretical contact with an underlying production function becomes only hypothetical. Instead, we shall here be concerned with an economic and social performance 'function' in a much wider sense.

Extending DEA in this manner, from a production function in the narrow sense to more generalized performance calculations, has long been standard in the DEA literature. This is exemplified by early DEA applications involving the rating of schools, hospitals and other not-for-profit DMUs (for references, see Charnes and Cooper [7]). The novelty in our application rests instead with the fact that these generalized production relationships for an entire nation are no longer determined by inherent input–output mechanisms but rather by economic policy. There exists no natural trade-off for a nation between productivity and, say, social policy. If such a trade-off does exist, it is all a matter of economic politics, the legislative bodies in the nation ascribing higher priorities to social policy than to productivity policy (being willing, say, to expand the social safety network of workers even it would have to be paid for by lowering real wages).

The first and immediate result of the DEA calculations is an efficiency rating of each observation (here, country). A rating of 100% indicates that the country is located on the efficiency frontier, tracing an optimal trade-off between economic and social factors. However, an efficiency rating of less than 100% signals non-optimal behavior. The performance of the nation is then non-Pareto optimal in the sense that, given the outputs, it should be possible to reduce the use of each input empolyed in the production process, both economic and social inputs.

A second set of calculations provides a measure of the 'returns of scale' of each country. In the well-known manner, constant RTS are said to prevail at a point on the frontier if an increase of all inputs by 1% leads to an increase of all outputs by 1%. But remember that both inputs and outputs now include social variables as well as economic ones! Decreasing RTS are present if outputs increase by less than 1%, while increasing RTS exist if they increase by more than 1%. Intuitively, then, decreasing RTS would be associated with a 'mature' economy where basic economic and social needs have already been covered, so that the incremental return of additional efforts is falling. Conversely, increasing RTS would seem to be associated with a developing nation with multiplying incremental returns on economic and social efforts.

For more than a decade, the mathematics of the calculations of RTS in DEA have been clouded by uncertainties, because of the possible presence of alternate optima and attendant mathematical problems. Fortunately, these difficulties have recently been resolved (see Banker and Thrall [8], Banker, Chang and Cooper [9], Golany and Yu [10] and Thore [11]). In the calculations presented below, we determine the coefficient of RTS for each nation. In the case of alternate optima, the entire range of the coefficient is listed. Furthermore, the calculations are carried out both in the context of the so-called BCC model [12] and the CCR model [3]. We believe that ours is the most extensive applied study of returns of scale yet to be reported in the DEA literature.

The calculations of returns of scale have a direct interpretation in terms of economic and social policy. In an obvious sense, a country with decreasing RTS has pushed economic and social policy too far, and one should expect political groupings to argue for a down-scaling of the social welfare ambitions. Conversely, a country with increasing RTS can be expected to be engaged in rapid economic and social growth. That is, these calculations tell us something about the long term growth of the country, and the direction it may take education, health, and social policy.

All standard methods of determining RTS proceed by examining tangential planes to the frontier that can be drawn through a given point. This is done either by looking at the constant term that represents the intercept of that plane with the plane in which all inputs are set to zero or, by observing the weights of the corner points of the facet of the frontier associated with that plane.

The determination of RTS is difficult because this plane need not be unique. But, fundamentally, the problem at hand is even more intriguing because the RTS may change when small displacements are affected by the data of the observed point. Therefore, we propose in this paper to investigate the RTS behavior in the neighborhood of the observed point where the investigation will be done along two directions: those of scale expansion and scale contraction.

The next section describes the data, lists the input–output factors employed, and reports on the countries that were fully efficient. We then discuss the measurements of RTS, and report on results for those inefficient countries that exhibit decreasing RTS, as well as those that exhibit increasing RTS. We then expand the analysis by testing for the RTS in two specified directions—to the 'left' and the 'right'—of each country located on the CCR-efficiency frontier. We close by summarizing our findings and offering some additional remarks on both the application and the methodology used in the paper.

PRELIMINARIES: DATA, DEA CALCULATIONS, AND A LISTING OF CCR-EFFICIENT COUNTRIES

Here we report on the estimation of a cross-country production frontier featuring not just the conventional economic inputs and outputs, but more general socio-economic indicators as well. A plausible scenario in which such an application would be considered is an evaluation by the World Bank or some U.N. agency of loan requests made by (one or more) developing countries. Such evaluations typically go beyond economic conditions in the narrow sense and include aspects of social circumstances, political stability, etc. That is, the concepts of inputs and outputs are now no longer limited to tangibles such as resources used or manufactured products obtained, but extend to social intangibles as well, such as education, crime prevention, public health, and so on.

The data were drawn from a rather remarkable data base on the economic and social performance of 118 countries assembled by Barro and Wolf [13] and distributed by the National Bureau of Economic Research. For the present purpose, we limited ourselves to data on 74 countries (=DMUs in this analysis) comprising the 'small sample' studied by Barro [14]. The sample includes poor, developing and developed countries from various regions and continents. The following input–output factors were employed:

Y_1 = growth rate of per capita Gross Domestic Product, average from 1970 to 1985;
Y_2 = 1 minus infant mortality rate ages 0–1, in 1985;
Y_3 = enrollment ratio for secondary education, in 1985;
Y_4 = ratio of nominal social insurance and welfare payments to nominal GDP, average from 1970 to 1985;
X_1 = ratio of real domestic investment to real GDP, average from 1970 to 1985;
X_2 = ratio of real government consumption expenditure net of spending on defense and on education, to real GDP, average from 1970 to 1985, and
X_3 = ratio of government expenditure on education to nominal GDP, average from 1970 to 1985.

The general idea behind this selection of variables is that as a country develops technologically and economically, it will also advance along a socio-economic scale—offering better social benefits to its citizens (more social insurance, better health, and better education, etc.).

All the data items are given as ratios. Normally, we would recommend using absolute physical values and not ratios (which may confuse inputs with outputs), but our hands were 'forced' owing to data availability. Hopefully, since most of the factors were divided by the GDP, potential size effects of the various national economies have been eliminated from the analysis.

For two countries, South Africa and Uganda, there was no information for the Y_4 variable; consequently, we dropped these countries from the sample and were left with a total population of 72 DMUs. Several data items required special care. Some of the output values (e.g. for Y_1) were negative. But DEA is quite capable of handling negative outputs (see, e.g. Refs [15] and [16]). The survival rate (Y_2) cannot exceed one by definition. Hence, a separate constraint was appended to the model formulation to account for this restriction (see Golany and Thore [17]).

For the DEA, we employed the basic CCR model (see Charnes, Cooper, Rhodes [3]), as follows:

$$\text{Min } \theta_0 - \epsilon \cdot \left[\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right] \tag{1}$$

s.t.

$$\sum_{j=1}^{n} Y_{rj} \cdot \lambda_j - s_r^+ = Y_{r0}, \quad r = 1, \ldots, s \tag{2}$$

$$\theta_0 \cdot X_{i0} - \sum_{j=1}^{n} X_{ij} \cdot \lambda_j - s_i^- = 0, \quad i = 1, \ldots, m \tag{3}$$

$$\lambda_j, s_i^-, s_r^+ \geqslant 0 \tag{4}$$

This program rates a particular DMU ($j = 0$). The unknowns are the weights λ_j to be attached to each DMU, the output and input slacks, s_j^+, s_j^-, respectively, and the desired efficiency rating θ_0 indicating the possible degree of contraction in all of DMU_0's inputs. The data parameters for the model are Y_{rj} and X_{ij}, the rth output and ith input of DMU_j, respectively, and ϵ, a small (infinitesimal) number. Using conventional terminology, we shall refer to the weighted sums

$$\sum_{j=1}^{n} Y_{rj} \cdot \lambda_j, \quad r = 1, \ldots, s \tag{5}$$

as 'best practice outputs', and to

$$\sum_{j=1}^{n} X_{ij} \cdot \lambda_j, \quad i = 1, \ldots, m \tag{6}$$

as 'best practice inputs'. In simple words, program (1)–(4) determines the efficiency rating θ_0 as the maximal possible rate of equi-proportional contraction of all inputs, while ensuring that best practice outputs do not fall short of observed outputs (2) and that best practice inputs do not exceed contracted inputs (3).

Note that the convexity constraint

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{7}$$

can be appended to model (1)–(4). This makes the model correspond to the BCC formulation of DEA (see Banker *et al.* [12]) rather than the CCR.

For the BBC model, any convex combination of the given observations is a feasible production point. For an illustration, see Fig. 1 where the case of a single input and a single output is shown. There are six observations, A, B, C, D, E and F. The piecewise linear frontier A–B–C–D is the BCC frontier while the four observations A, B, C, and D are BCC-efficient. But, an observation like E is inefficient and best BCC practice for E is the projection E′ on AB. Similarly, point F′ can be obtained as a convex combination of the corner points C and D.

The CCR model satisfies the following 'ray property': if (X, Y) is a feasible production point, then (kX, kY) is also a feasible point, where k is a nonnegative scalar.

The ray O–B–M is the CCR frontier for this example. Observation B is CCR-efficient. All other observations are CCR inefficient. Best CCR practice for F is the projection F″ on O–B–M. The CCR frontier exhibits constant RTS throughout. The BCC frontier exhibits increasing RTS along A–B, and decreasing RTS along B–C–D.

Turning now to the empirical results, we first list all countries that were CCR efficient (like point B in Fig. 1), and thus exhibit constant RTS. It turned out that 23 countries (out of the

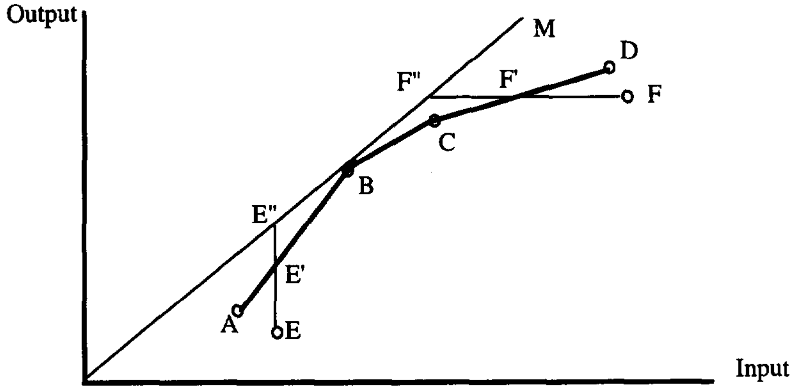

Fig. 1. CCR and BCC efficiency illustrated.

population of 72 countries) obtained a CCR-efficiency rating equal to one. These are the countries at the frontier of achievement in terms of both economic growth and social policy. These results are listed in Table 1.

Inspecting the list of countries, note that it includes both Japan and three (of the four) 'Asian Tigers' of the Far East: Korea, Taiwan, and Singapore (Hong Kong was not included in this sample). The list includes both Canada and the U.S. It also includes four African countries and five Latin American countries.

THE MATHEMATICS OF THE RETURNS TO SCALE CALCULATIONS

Program (1)–(4), (7) is usually referred to as an 'envelopment' formulation of DEA. We also write down the corresponding 'multiplier' formulation, i.e. the corresponding dual program. This formulation makes use of the virtual multipliers of all outputs μ_r, $r = 1, \ldots, s$ and the virtual multipliers of all inputs v_i, $i = 1, \ldots, m$. It also makes use of the dual to (7), say u_0. The program reads as follows:

$$\max \sum_{r=1}^{s} Y_{r0} \cdot \mu_r - u_0 \tag{8}$$

Table 1. Results for countries located at the CCR efficiency frontier

Country	CCR efficiency	Max u_0	Min u_0	Max $\Sigma_j \lambda_j$	Min $\Sigma_j \lambda_j$
Ghana	1.000	3.623	−1.062	1.000	1.000
Morocco	1.000	4.660	−0.880	1.000	1.000
Tunisia	1.000	4.436	−0.590	1.000	1.000
Zaire	1.000	4.436	−0.918	1.000	1.000
Japan	1.000	9999	−0.320	1.000	1.000
Korea	1.000	9999	−0.263	1.000	1.000
Pakistan	1.000	1.244	−1.000	1.000	1.000
Singapore	1.000	9999	−1.000	1.000	1.000
Taiwan	1.000	9999	−0.080	1.000	1.000
Belgium	1.000	9999	−0.630	1.000	1.000
France	1.000	799.9	−0.433	1.000	1.000
Luxembourg	1.000	28.48	−0.442	1.000	1.000
Malta	1.000	9999	−0.131	1.000	1.000
Netherlands	1.000	9999	−0.802	1.000	1.000
Spain	1.000	463.9	−0.835	1.000	1.000
Switzerland	1.000	9999	−1.000	1.000	1.000
Canada	1.000	9999	−0.068	1.000	1.000
El Salvador	1.000	14.96	−1.068	1.000	1.000
Guatemala	1.000	13.20	−1.000	1.000	1.000
U.S.	1.000	39.87	−0.564	1.000	1.000
Paraguay	1.000	15.84	−1.000	1.000	1.000
Peru	1.000	1.372	−0.691	1.000	1.000
Uruguay	1.000	14.03	−0.736	1.000	1.000

s.t.

$$\sum_{i=1}^{m} X_{i0} \cdot v_i = 1 \tag{9}$$

$$\sum_{r=1}^{s} Y_{rj} \cdot \mu_r - \sum_{i=1}^{m} X_{ij} \cdot v_i - u_0 \leqslant 0, \quad j = 1, \ldots, n \tag{10}$$

$$-\mu_r \leqslant -\epsilon, \quad r = 1, \ldots, s \tag{11}$$

$$-v_i \leqslant -\epsilon, \quad i = 1, \ldots, m \tag{12}$$

Starting out from relation (10), consider

$$\sum_{r=1}^{s} y_r \cdot \mu_r - \sum_{i=1}^{m} x_i \cdot v_i - u_0 \leqslant 0 \tag{13}$$

where the y_r, x_i are now variables rather than given observations. Equation (10) represents a supporting hyperplane for the production possibility set. All observations lie on one side of it (as indicated by the $\leqslant$ sign in (10)). The value of u_0 indicates the intercept at the origin.

Consider any point along the facet B–C in Fig. 1. The supporting hyperplane is B–C itself. The intercept is positive, and there are decreasing RTS. Or, consider any point along the facet A–B. The intercept of the supporting hyperplane is negative, and there are increasing RTS. If the intercept were zero, there would be constant RTS.

However, when multiple inputs and outputs are considered, the supporting hyperplane is often not uniquely determined. Consider point C in Fig. 1. The supporting hyperplane at point C is any straight line through C whose slope is at least as steep as that of C–D but not steeper than that of B–C. In any case, the intercept is positive, and there are decreasing RTS. However, at point B the RTS are indeterminate and depend on whether we wish to associate the RTS with facet B–C or A–B.

The possible indeterminacy now discussed arises because the linear programs can have alternate optima. To investigate these possibilities, we now propose to calculate the interval (min u_0, max u_0). If this interval is entirely positive, there are decreasing RTS. If it is entirely negative, there are increasing RTS. But, if the interval straddles both positive and negative values, then the RTS are indeterminate.

Min u_0 will be solved by the program

$$\text{Min } u_0 \tag{14}$$

s.t.

$$\sum_{r=1}^{s} Y_{r0} \cdot \mu_r - u_0 \geqslant \left(\sum_{r=1}^{s} Y_{r0} \cdot \mu_r - u_0 \right)^* \tag{15}$$

$$\sum_{i=1}^{m} X_{i0} \cdot v_i = 1 \tag{16}$$

$$\sum_{r=1}^{s} Y_{rj} \cdot \mu_r - \sum_{i=1}^{m} X_{ij} \cdot v_i - u_0 \leqslant 0, \quad j = 1, \ldots, n \tag{17}$$

$$-\mu_r \leqslant -\epsilon, \quad r = 1, \ldots, s \tag{18}$$

Table 2. Intercept u_0 as a criterion of RTS

Returns to scale	Criterion
Decreasing RTS	min $u_0 > 0$
Constant RTS	CCR-efficiency = 1
Increasing RTS	max $u_0 < 0$

Table 3. Sum of weights $\Sigma_j\lambda_j$ as criterion of RTS

Returns to scale	Criterion
Decreasing RTS	min $\Sigma_j\lambda_j > 1$
Constant RTS	CCR-efficiency = 1
Increasing RTS	max $\Sigma_j\lambda_j < 1$

$$-v_i \leqslant -\epsilon, \quad i = 1, \ldots, m \tag{19}$$

where the asterisk (*) denotes the optimal value already determined in solving (8)–(12). In the same fashion, Max u_0 is determined by maximizing u_0 subject to the same constraints (15)–(19). (For an earlier reference to such 'dual' optimizations in the RTS context, see Banker and Thrall [8].)

Returning to Table 1, note that in every single case the intercept u_0 was non-unique. This should not surprise since efficient observations typically correspond to corner points along the frontier.

The results discussed now are summed up in Table 2.

An alternative approach to determining the RTS is to examine the sum of weights $\Sigma_j\lambda_j$ determined in the CCR model. If the weights add up to more than unity, the best practice point (5) and (6) not only involves a convex combination of the observations but also a radial expansion. In other words, the scale of operation of best practice is larger than the convex combination at which there would be constant RTS. To achieve constant RTS, the inputs and outputs of the given observation have to shrink. There are then locally decreasing RTS.

However, if the sum of weights $\Sigma_j\lambda_j$ adds up to less than unity, the best practice point not only involves a convex combination of the observations but also a radial contraction. The scale of operation of best practice is smaller than the convex combination at which there would be constant RTS. To achieve constant returns, the inputs and outputs of the given observation would have to expand. There are then locally increasing RTS.

In their original 1984 paper [12], Banker, Charnes and Cooper showed that if the DEA calculation yields a unique optimum, the u_0 and the $\Sigma_j\lambda_j$ criteria provide consistent information, as follows:

decreasing RTS	$u_0 > 0,$	$\Sigma_j\lambda_j > 1$
constant RTS	$u_0 = 0,$	$\Sigma_j\lambda_j = 1$
increasing RTS	$u_0 < 0,$	$\Sigma_j\lambda_j < 1$

However, if there are alternate optima, things become more complicated. We have already explored the possibility that u_0 is non-unique. It thus remains to discuss the case that $\Sigma_j\lambda_j$ is non-unique. To investigate this matter, we shall determine the interval (min $\Sigma_j\lambda_j$, max $\Sigma_j\lambda_j$). If this interval in its entirety is greater than one, there are decreasing RTS. If it is less than one in its entirety, there are increasing RTS. But, if the interval straddles both values greater than one and less than one, the RTS are indeterminate.

For this purpose, we solve

$$\text{Min} \sum_{j=1}^{n} \lambda_j \tag{20}$$

s.t.

$$\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \geqslant \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)^* \tag{21}$$

$$\sum_{j=1}^{n} Y_{rj}\cdot\lambda_j - s_r^+ = Y_{r0}, \quad r = 1, \ldots, s \tag{22}$$

$$\theta_0 \cdot X_{i0} - \sum_{j=1}^{n} X_{ij} \cdot \lambda_j - s_i^- = 0, \quad i = 1, \ldots, m \tag{23}$$

$$\lambda_j, s_i^-, s_r^+ \geqslant 0 \tag{24}$$

where the asterisk (*) denotes the optimal value already determined in solving (1)–(4).

In the same fashion, Max $\Sigma_j \lambda_j$ is determined by maximizing $\Sigma_j \lambda_j$ subject to the same constraints (21)–(24).

For all countries listed in Table 1, we found min $\Sigma_j \lambda_j$ = max $\Sigma_j \lambda_j = 1$, confirming that these countries all exhibit constant RTS.

Again, we sum up our results in Table 3.

SLOWING DOWN A NATION'S ECONOMIC AND SOCIAL EXPANSION: THE CASE OF DECREASING RETURNS TO SCALE

In this section, we report on the efficiency and RTS results for those countries that (i) were found to be less than 100% CCR efficient, and, at the same time, (ii) exhibited decreasing RTS. Inefficiency means that it would be possible for a nation to reduce the use of its inputs X while still obtaining the same amounts (or more) of the outputs Y. Looking at point F in Fig. 1 again, it would be possible to reduce inputs by the amount F–F″.

RTS for an inefficient observation refers to the RTS of its projection on the BCC-efficiency frontier. In the case of point F, it refers to the returns of scale at point F′.

There is no reason to believe that an inefficient observation such as F will tend to improve its inefficiency over time, advancing toward point F′. There will no doubt be calls for improved productivity, industry modernization, and improved organization of education and the health system. Inefficiency is often deeply embedded in existing economic and social structures, like weak entrepreneurial spirits, poor functioning of capital markets, disincentives created by tax codes, lack of modern equipment in the health sector, poor nutrition, and so on. Such structures are difficult to change, and may be subject only to indirect government control.

We shall, however, make the assumption that the presence of decreasing RTS (at point F′) will lead to political pressure to halt the growth in government spending on education, health, and social welfare. Voices will be heard proclaiming that such spending has been pushed 'too far' and that the country can no longer 'afford' the generous social policies of the past.

We characterize countries in this position as being 'mature'. As we shall now see, the countries belonging to this category are not just many developed nations, but also include several countries in the Third World. Our results are summarized in Table 4.

The list includes the U.K. and the Scandinavian countries, which often have been admired for their educational system and extensive social welfare. Yet, the efficiency ratings of some of these countries are quite low (0.75 for Norway and 0.77 for Denmark). On the other side of the globe, Australia and New Zealand did not fare any better.

Our data refer to the time period 1970–1985. It should come as no surprise that in all these countries there has recently been strong political pressure to halt the growth of the welfare state.

Some very poor countries also belong to the same category, such as Botswana, Burma, Panama, Guyana, Fiji and New Guinea. It is unfortunate enough that these countries display low efficiency ratings. Even more discouragingly, they also exhibit falling returns of scale. These countries desperately need improved educational resources and a social welfare net. And yet, such an expansion of educational and welfare policy would only offer, at least in the short-term future, falling marginal returns.

THE REWARDS OF EXPANSION: THE CASE OF INCREASING RETURNS TO SCALE

We now turn to those countries that (i) were found to be less than 100% CCR efficient, and, at the same time, (ii) exhibited increasing RTS. To illustrate such a country, consider point E in Fig. 1.

It is easy to argue that rather than solving the 'input-oriented' DEA program (1)–(4), one should now solve an 'output-oriented' formulation aimed at expanding all outputs equi-proportionally. The resulting efficiency rating will be the same, but 'best practice', that is, the projection on the efficiency frontier, will be different. Point E will then be projected on point E′ on the BCC frontier and onto point E″ on the CCR frontier.

In any case, we shall make the assumption that the presence of increasing RTS (such as at point E′) will spur a political pressure to step up the growth in government spending on education, health, and social welfare. Such spending will be seen as sound investment in a productive workforce and in human capital. Thus motivated, the country will follow a path of fiscal activism. The government sector will grow, and this expansion will be seen as economically motivated.

We characterize countries in this position as being 'young', where all are developing. The results are summarized in Table 5.

The list includes major Sub-Saharan African countries and Latin-American countries, as well as three European countries: Greece, Iceland, and Turkey.

RETURNS TO SCALE TO THE LEFT AND TO THE RIGHT

The purpose of this section is to shed further light on the results obtained for the countries listed in Table 1. In accordance with common practice, we have identified those countries as exhibiting 'constant RTS'. Returning to Fig. 1, we observe that there is indeed constant RTS along the frontier O–B–M. Yet, some readers may find it discomforting that the concept of 'constant' RTS is defined with reference to one frontier—the CCR frontier—while 'decreasing' and 'increasing' RTS are defined with reference to an entirely different frontier—the BCC frontier. The question then naturally presents itself (still looking at CCR-efficient points): how would we rate the RTS of such points with reference to the BCC frontier?

Tracing the BBC frontier A–B–C–D in Fig. 1 and looking at point B again, the RTS at this point are discontinuous, jumping from increasing RTS along A–B to decreasing RTS along B–C.

Table 4. Inefficient countries exhibiting both CCR-inefficiency and decreasing RTS

Country	CCR efficiency	Max u_0	Min u_0	Max $\Sigma_j\lambda_j$	Min $\Sigma_j\lambda_j$
Botswana	0.496	0.044	0.044	1.051	1.051
Cameroon	0.840	0.473	0.473	1.008	1.008
Egypt	0.993	0.145	0.145	1.045	1.045
Mauritius	0.913	7.632	3.586	1.026	1.026
Burma	0.937	1.134	1.134	1.045	1.045
India	0.632	0.129	0.129	1.022	1.022
Israel	0.796	3.432	3.432	1.009	1.009
Malaysia	0.896	1.888	1.888	1.018	1.018
Sri Lanka	0.925	2.042	2.042	1.017	1.017
Thailand	0.979	2.732	1.465	1.029	1.029
Austria	0.874	115.69	0.736	1.172	1.172
Cyprus	0.724	5.598	5.598	1.108	1.108
Denmark	0.777	9999	1.744	1.129	1.129
Finland	0.683	9999	3.311	1.096	1.096
Ireland	0.647	1.146	1.146	1.043	1.043
Italy	0.783	1.023	1.023	1.003	1.003
Norway	0.749	9999	4.061	1.176	1.176
Sweden	0.854	9999	27.24	1.009	1.009
U.K.	0.943	135.04	21.15	1.027	1.027
Barbados	0.958	1.734	1.734	1.120	1.120
Costa Rica	0.726	12.061	12.061	1.049	1.049
Panama	0.504	3.552	3.552	1.046	1.046
Chile	0.565	3.969	3.969	1.005	1.005
Colombia	0.904	0.365	0.365	1.003	1.003
Guyana	0.419	4.092	4.092	1.026	1.026
Venezuela	0.835	8.511	8.511	1.014	1.014
Australia	0.777	3.778	3.778	1.001	1.001
Fiji	0.651	2.324	2.324	1.009	1.009
New Zealand	0.679	8.131	8.131	1.039	1.039
New Guinea	0.480	0.285	0.285	1.007	1.007

Table 5. Inefficient countries exhibiting CCR-inefficiency and increasing RTS

Country	CCR efficiency	Max u_0	Min u_0	Max $\Sigma_j \lambda_j$	Min $\Sigma_j \lambda_j$
Kenya	0.513	−0.504	−0.504	0.978	0.978
Liberia	0.528	−0.559	−0.559	0.944	0.944
Malawi	0.670	−0.621	−0.621	0.933	0.933
Senegal	0.890	−0.870	−0.870	0.930	0.930
S. Leone	0.717	−0.826	−0.826	0.882	0.882
Zambia	0.334	−0.290	−0.290	0.982	0.982
Iran	0.785	−0.520	−0.520	0.927	0.927
Jordan	0.723	−0.375	−0.375	0.973	0.973
Philippines	0.899	−0.587	−0.587	0.984	0.984
Greece	0.877	−0.108	−0.108	0.933	0.933
Iceland	0.963	−0.258	−0.250	1.000	1.000
Turkey	1.000	28.48	−0.442	1.000	1.000
Dominican R.	0.947	−0.806	−0.806	0.965	0.965
Mexico	0.849	−0.643	−0.643	0.980	0.980
Nicaragua	0.769	−0.580	−0.580	0.985	0.985
Argentina	0.930	−0.660	−0.660	0.979	0.979
Bolivia	0.718	−0.567	−0.567	0.942	0.942
Brazil	0.830	−0.442	−0.442	0.969	0.969
Equador	0.679	−0.232	−0.232	0.988	0.988

Or, with an obvious expression, the RTS to the left of point B are increasing, while the RTS to the right of point B are decreasing.

The situation depicted in Fig. 1 is only one of many possible scenarios. Suppose, for instance, that point C happened to be located on the ray O–B, so that the segment B–C exhibited constant RTS. Then, the RTS to the left of point B would still be increasing, but the RTS to the right of point B would be constant.

In order to investigate these matters more systematically, we use the model formulations developed by Golany and Yu [10], as follows:

$$\text{Min } \theta_0 - \epsilon \cdot \left[\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right] \tag{25}$$

s.t.

$$\sum_{j=1}^{n} Y_{rj} \cdot \lambda_j - s_r^+ = (1 + \delta) \cdot Y_{r0}, \quad r = 1, \ldots, s \tag{26}$$

$$\theta_0 \cdot X_{i0} - \sum_{j=1}^{n} X_{ij} \cdot \lambda_j - s_i^- = 0, \quad i = 1, \ldots, m \tag{27}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{28}$$

$$\lambda_j, s_i^-, s_r^+ \geqslant 0 \tag{29}$$

where δ is a small positive number. In the numerical calculations to follow, we shall use $\delta = 0.01$. We recognize program (25)–(29) as the standard BCC program, where the outputs of DMU_0 have been slightly perturbed upwards (each output is multiplied by the factor $(1 + \delta)$). The optimal θ_0^* achieved from this program then yields information about the RTS to the right of the given observation, as listed in Table 6.

Next, we solve the program

$$\text{Max } \theta_0 + \epsilon \cdot \left[\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right] \tag{30}$$

Table 6. Returns to scale to the right of DMU_0

Value of θ_0^*	Returns to scale to the right of DMU_0
$(1+\delta) > \theta_0^* > 1$	Increasing
$1 \geqslant \theta_0^*$	DMU_0 is BCC-inefficient
$(1+\delta) = \theta_0^*$	Constant
$(1+\delta) < \theta_0^*$	Decreasing
No feasible solution for θ_0^*	Undetermined

Table 7. Returns to scale to the left of DMU_0

Value of θ_0^*	Returns to scale to the left of DMU_0
$(1-\delta) < \theta_0^* < 1$	Decreasing
$1 \leqslant \theta_0^*$	DMU_0 is BCC-inefficient
$(1-\delta) = \theta_0^*$	Constant
$(1-\delta) > \theta_0^*$	Increasing
No feasible solution for θ_0^*	Undetermined

s.t.

$$\theta_0 \cdot Y_{r0} - \sum_{j=1}^{n} Y_{rj} \cdot \lambda_j + s_r^+ = Y_{r0}, \quad r = 1, \ldots, s \tag{31}$$

$$\sum_{j=1}^{n} X_{ij} \cdot \lambda_j + s_i^- = (1-\delta) \cdot X_{i0}, \quad i = 1, \ldots, m \tag{32}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{33}$$

$$\lambda_j, s_i^-, s_r^+ \geqslant 0 \tag{34}$$

We recognize program (30)–(34) as a standard output-oriented BBC program, where all the inputs of the given observation have been slightly perturbed downward (each input is multiplied by the factor $(1-\delta)$). The optimal θ_0^* obtained from this program then yields information about the RTS to the left of the given observation as exhibited in Table 7.

All empirical results now obtained are summarized in Table 8. Inspecting the table, we see that in five cases (Morocco, Tunisia, Zaire, Peru and Uruguay) the outcomes of the analysis conform with the expected behavior of a unit exhibiting constant RTS—namely, increasing RTS to the left and decreasing RTS to the right (as illustrated by point B in Fig. 1).

The measure of RTS in DEA is a local one. It holds for the DMU being evaluated in its current position. Occasionally, even minute changes in this position may change the direction of RTS indicated by DEA (see a discussion of this property in Banker *et al.* [18, p. 145]). This inherent

Table 8. Further results for countries located at the CCR efficiency frontier

	Returns to scale on the left		Returns to scale on the right	
Country	θ_0^* by (25)–(29)	RTS	θ_0^* by (30)–(34)	RTS
Ghana	Infeasible	Undetermined	1.046	Decreasing
Morocco	0.916	Increasing	1.057	Decreasing
Tunisia	0.976	Increasing	1.052	Decreasing
Zaire	0.877	Increasing	1.054	Decreasing
Japan	0.985	Increasing	Infeasible	Undetermined
Korea	0.986	Increasing	Infeasible	Undetermined
Pakistan	Infeasible	Undetermined	1.022	Decreasing
Singapore	0.991	Decreasing	Infeasible	Undetermined
Taiwan	0.994	Decreasing	Infeasible	Undetermined
Belgium	0.997	Decreasing	Infeasible	Undetermined
France	0.983	Increasing	Infeasible	Undetermined
Luxembourg	0.985	Increasing	Infeasible	Undetermined
Malta	0.9898	Increasing	Infeasible	Undetermined
Netherlands	0.9945	Decreasing	Infeasible	Undetermined
Spain	0.940	Increasing	Infeasible	Undetermined
Switzerland	Infeasible	Undetermined	Infeasible	Undetermined
Canada	0.9904	Decreasing	Infeasible	Undetermined
El Salvador	Infeasible	Undetermined	1.160	Decreasing
Guatemala	Infeasible	Undetermined	1.142	Decreasing
U.S.	0.988	Increasing	Infeasible	Undetermined
Paraguay	Infeasible	Undetermined	1.168	Decreasing
Peru	0.969	Increasing	1.012	Decreasing
Uruguay	0.972	Increasing	1.150	Decreasing

Table 9. Summary of RTS to the left and to the right

To the right / To the left	IRT $1 \leqslant \theta < 1.01$	CRT $\theta = 1.01$	DRT $\theta > 1.01$	Undetermined θ infeasible
IRT $0 < \theta < 0.99$	—	—	Morocco Tunisia Zaire Peru Uruguay	Japan France Luxembourg Malta Spain U.S.
CRT $\theta = 0.99$	—	—	—	—
DRT $1 > \theta > 0.99$	—	—	—	Korea Singapore Taiwan Belgium Netherlands Canada
Undetermined θ infeasible	—	—	Ghana Pakistan El Salvador Guatemala Paraguay	Switzerland

instability makes the analysis to the left and to the right of DMU_0 a crucial element in any study of RTS.

Returning to Table 8, we find that all other countries present an apparent instability in the RTS characterization. Although they were all earlier denoted as CCR-efficient (cf. Table 1), when their immediate neighborhood was investigated (through models (25)–(29) and (30)–(34)), we now find that they must actually be associated with an area of the efficiency frontier where no feasible solution for θ_0 can be found when they are tested either to the left (Ghana, Pakistan, El Salvador, Guatemala and Paraguay) or to the right (Korea, Singapore, Taiwan, Belgium, Netherlands and Canada).

As explained in Golany and Yu [10], such phenomena imply that these observations reside on either a 'left-most' or 'right-most' corner of the frontier, respectively. This concept is visually easy to comprehend by looking at points A and D in Fig. 1. When we attempt to move to the left of A the model is unable is unable to envelop the analyzed point since there are no observations left of point A. The same is happening, in the reverse direction, when we attempt to move to the right of point D. In a multi-dimensional situation, it is more difficult to determine exactly what point is at the left-most or right-most corners and, in fact, there could be several such points in each of the two 'corners' of the frontier.

The results just described demonstrate the necessity to analyze the RTS characterization of DMUs in a more accurate manner than has previously been the practice.

Finally, we find it useful to summarize the overall RTS information now obtained in the 4×4 format shown in Table 9. As it turns out, only 5 of 16 cells contain entries. Were some other data set evaluated, however, one would no doubt find observations associated with other cells in the table. Thus, even the most undesirable situation can occur—that the RTS are undetermined in both directions—in fact, it occurs for Switzerland (the same indeterminacy would obtain, were the data set to contain just one single observation).

CONCLUDING REMARKS

In this paper, we have evaluated the economic and social performance of 72 countries during the time period 1970 to 1985. Performance was measured in terms of both economic and social performance (the growth of GDP per capita, the infant mortality rate, enrollment in secondary education, and social insurance and welfare payments). The inputs used to assess the performance of each country were domestic investments, and government expenditures on both economic and social programs.

A novel feature of our study is the systematic and detailed calculation of the rate of RTS for each country. On the basis of our results, the 72 countries were divided into three groups:

(i) CCR-efficient countries, exhibiting local constant RTS. This list includes Japan and three of the 'Tigers' of the Far East. It may surprise the reader that these countries do not only exhibit outstanding growth but also superior achievements in terms of education and social policy. Canada and the U.S. also belong to this group.

(ii) CCR-inefficient countries, exhibiting decreasing RTS and presumably being engaged in efforts to limit the expanding social welfare sector of the economy. We have referred to these countries as being 'mature', having pushed education, health and social policy to the point where the marginal returns on such efforts are declining. The list includes the U.K. and the Scandinavian countries, Australia and New Zealand. Some very poor countries also belong to the same category.

(iii) CCR-inefficient countries, exhibiting increasing RTS and presumably being engaged in efforts to grow the social welfare sector of the economy. We have referred to these countries as being 'young'. They are all developing countries and for them investment in education, health and welfare is a profitable proposition in that the marginal returns of such investments are increasing.

Until recently, calculations of rates of return have been marred by the mathematical possibility of alternate optima in DEA calculations, so that the resulting measure of RTS can be non-unique. Fortunately, the attendant mathematical problems have now been clarified and the possibility of alternate optima in two measures of rates of return have been examined: the intercept of the supporting enveloping hyperplane (the coefficient u_0), and the sum of weights to be attached to the reference observations (the sum $\Sigma_j \lambda_j$). Both measures can be non-unique, so that a possibility of ambiguity can arise on two different fronts.

To deal with these matters, we have, for each country, calculated both measures of RTS; in the case of non-uniqueness, the entire range of possible alternate optima was determined. Our results can be summarized as follows:

Alternate optima for the sum of weights $\Sigma_j \lambda_j$ in the present study are quite rare. We did not encounter a single case of alternate optima in this sense. (However, other studies have reported the presence of alternate optima, see e.g. Thore [11].)

All CCR-efficient observations have exhibited alternate optima expressed in alternate signs for intercept (u_0) of the supporting hyperplane. Further investigation of the sensitivity of the constant RTS outcome for these countries revealed that some of them were indeed associated with slopes of the frontier which were increasing on their left and decreasing to their right. For the majority of CCR-efficient countries, however, the constant RTS outcome was found to be highly sensitive to small changes in one or both of the directions tested.

For all other observations, alternate optima, in the sense of the intercept of the supporting hyperplane, are rare. We did not encounter any such observations.

These propositions will not hold in general, of course. In very large data sets, ambiguities will be encountered, but they are likely to be rare.

The sensitivity analysis obtained through the tests of RTS to the left and to the right can obviously be expanded to encompass any possible direction around the given observation DMU_0 and, in particular, along the directions that are associated with changes of individual inputs or outputs. The investigation of such 'directional RTS' is clearly a subject for additional research, which the authors plan to undertake.

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