SIMULTANEOUS DETERMINATION OF VISITS TO A SYSTEM OF OUTDOOR RECREATION PARKS WITH CAPACITY LIMITATIONS

Mira BARON and Mordechai SHECHTER*

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Summary: Network tools are used in computing demand for recreational services.

1. Introduction

The planning process in outdoor recreation involves two main functions. One is the projection of future aggregate demand levels for the different outdoor recreation activities. The other is the prediction of the spatial distribution of these demands. The solution to the planning problem lies therefore not only in the provision of sufficient facilities (at prices people are willing to pay, individually, or through government subsidies and grants), but also in the location and timing of that provision.

The two common methodological approaches to the problem of outdoor recreational planning are (a) econometric planning models and (b) physical planning models. Econometric models attempt to derive statistical estimates, usually in the form of demand functions, for recreational facilities in time and space. This approach which originated with Clawson (1959) and Knetsch (1963), focuses on estimating a demand function for a given park, based on differentials in visit rates to that park from various population centers. It was expanded and generalized by Burt and Brewer (1971) to obtain a set of simultaneous

* Respectively, Research Associate at the Center for Urban and Regional Studies, and Senior Lecturer at the Faculty of Industrial and Management Engineering, Technion – Israel Institute of Technology, Haifa, Israel. The authors wish to thank Messrs. Y. Cohen of Economic Consulting and Planning Ltd., Z. Unger of the Jewish Agency for Israel for permission to use the data from the Judean Hills recreational survey. We wish also to thank Professor Jack L. Knetsch for his comments on an earlier draft.
demand equations to a system of outdoor recreation parks. Customarily, these studies assume that the factors affecting demand, such as price, income, and consumers' preferences, are independent of those affecting supply, such as the number of outdoor recreation parks.

The physical approach to recreational planning involves the prediction of an equilibrium point in the price-quantity space. Rather than calculating a demand schedule, it determines the point of intersection between supply and demand. With the exception of two studies (Seneca et al., 1968; Cicchetti et al., 1969) these models are patterned on the behaviour of physical systems. Their basic postulate is that the behavioural relationships of the recreational system are similar, if not identical, to their relevant counterparts in a physical analogue.

The most popular physical model is the gravitation model. It was first applied to predicting visits to a given recreational site, as in the Clawson model. An expanded version of the gravitation model (Cesario, 1969; Wennegren and Nielsen, 1970) attempts to explain the spatial distribution of recreationists in a system of parks, at a given point in time (e.g., weekend). The major factor affecting the distribution is the relative attraction value of each of the parks. A related approach applied to a system of parks is that of the intervening opportunities model (Clark and Roeske, 1969; Thompson, 1969).

Another physical approach was first suggested by Ellis (1966) and Ellis and Van Doren (1966). In this model, called RECSYS, the outdoor recreation system of parks is patterned after an electric network, using linear systems theory. Here, the spatial distribution of recreationists is a function of the road network as well as of the parks' characteristics.

In the present work we propose some modifications in the electrical network approach which we believe improve its realism and applicability as a planning tool. We also attempt to narrow the gap between the econometric and physical modelling approaches. We propose to do this by first describing the econometric and physical approaches in some detail (sect. 2), then moving on to the basic concepts in electric circuit theory and their relevance to the outdoor recreation system (sect. 3). This will clarify the use of the electrical network model and the suggested modifications. Sect. 4 follows with a description and critical appraisal of the RECSYS model and describes some proposed modifications. These modifications are the addition of a dynamic representation of park attendance during a typical weekend as well as of parks' capacities, and an attempt to derive demand
functions for the parks in the system. Finally, sect. 5 describes an application of the modified model to data from a survey of recreationists in the Judean Hills recreation region of Israel.

2. Econometric and physical models

2.1. Econometric models

The underlying assumption in the estimation of a demand schedule is that recreation is a good like any other good in which most, if not all, of the consumption benefits accrue to the user himself. Any externalities in consumption are assumed to be negligible. Another basic assumption is that the benefit which a person derives from a recreation facility is related to the distance which he is prepared to travel in order to reach it and hence related to the cost of travel.

In addition to this travel item, which expresses the monetary value of the disutility of overcoming distance, it is necessary to include additional items such as outlay for food and lodging (over and above normal expenses), entrance fees, and the opportunity costs of time (travel time plus time spent at the facility).

The leisure time constraint accounts for the positive monetary value of time – this time could have been spent in other leisurely pursuits with positive marginal utilities. Alternatively, the recreational time could have been spent earning additional wages. Distance involves some disutility associated with travel. However, it could yield pleasurable experience for some forms of travel, especially on some scenic roads. Usually, longer distances, or lower quality roads, or congested highways increase the degree of discomfort associated with travelling to an outdoor recreational facility. In this paper we adopt the accepted assumption of complementarity among the various cost items – tangible as well as intangible – associated with outdoor recreational activities. This simplifying assumption, probably valid in most instances, enables us to use distance as a proxy variable for total (pecuniary, time and discomfort) expenditure levels. We thus neglect the possibility for substitution among these items (e.g., using faster but more expensive means of transportation) and postulate that longer distances imply, inter alia, proportionately higher cost levels.

The Clawson–Knetsch approach to constructing a demand schedule for recreation also uses distance as a proxy variable for expenditure levels. Since there is no market for publicly owned and operated
outdoor recreational parks, and hence no mechanism for registering the value of their outputs, the various expenditure levels assume the meaning of prices. In this approach, all points of origin of visitors to the recreation site are grouped into distance zones around it, and average travel expenditures from each zone to the site are calculated. Next, the visit rate from each zone to the given site during a predetermined period is measured. The derived relationship between distance zones and their respective visit rates has been termed the demand for "total recreational experience" at the given site. This approach further assumes that populations in all zones are homogeneous with respect to their expected response to changes in costs, and hence changes in entrance fees could be construed as having the same effect on the quantity demanded by people coming from a more distant location to the park. By computing total number of visits expected at various levels of entrance fees, an approximation to the demand curve for the recreational facility is derived. From such a demand curve it would be possible to predict the number of visits to an existing facility resulting from changes such as reduction in travel costs, change in travel time due to improved roads, or differing levels of entrance fees.

This approach ignores three centrally relevant aspects. Firstly, it is not easily applicable to the problem of predicting demand for new recreational facilities. Secondly, it fails to consider the price of substitute products such as competing recreational areas in the immediate vicinity of population centers, between them and the given park, and in the vicinity of that park itself. Thirdly, it completely neglects the twin phenomena of road congestion and park crowding which yield increasingly widespread externalities.¹

Burt and Brewer (1971) suggest an interesting approach towards incorporating the effects of substitute sites and the prediction of new park attendance.² Their approach is to construct a system of simultaneous demand equations, each equation representing demand to a different type of recreational park. Their typology is based predominantly on the range of recreational activities which are most characteristic of any given park, such as water-oriented activities, hiking activities, etc.

They attempt to estimate demand schedules for recreational activities at various distances (viz., prices) from a single population center (rather

¹ An undated study by Hastings and Tolley is one of a few exceptions to that neglect.
² See also Mansfield (1971) who approached the problem of predicting attendance at a single new park in a similar manner.
than a single park and several population centers). The demand for any activity is expressed as a function of its own ‘price’, the income level of the population, and the price of substitute activities. Thus, a new park of a given recreational type, closer to the population center, reduces the price of the activity (typical to this park) and the overall structure of relative prices. The impact of such action would be to change attendance at other types of parks and, of course, at existing parks of the same type. The approach suggested in this paper is analogous to Burt and Brewer’s although the methodology is different.

2.2. Physical models

The analog physical approach attempts to find an analogy between a real social system and a physical model. For example, in this work the physical model is an electrical analog based on electric circuit theory. The modelled system is the demand and supply for outdoor recreation facilities. On the basis of this analogy, we apply the analytical and empirical results of the physical model to the analysis of the social system and by studying the behaviour of such a model we obtain insight into the behaviour of the modelled system. In the case of the analog model, the set of laws governing the behaviour of the physical model has a counterpart in the set of functional relationships characterizing the modelled system. Thus, the equations describing the functional behaviour of the electrical analog (such as Ohm’s Law and Kirchoff’s Laws) are used to represent equilibrium demand and supply relationships postulated for a system of outdoor recreation parks and population centers.

The most notable advantage of using an analog stems from its postulated close correspondence to a specific real system which is the object of the modelling process. Thus, the analytical model and the accompanying solution procedures, such as linear systems theory in the case of electrical networks, are readily available for the purpose of analyzing a complex real system.

A physical model frequently found in the outdoor recreation planning literature is the gravity model. It is a formal expression of Newton’s Law of Gravity. The gravity formula expresses the trip volume between a population center and a given park in terms of the distance (or travel time) between them and the size of the population at the point of origin. The simple gravity model handles the problem of substitution among parks by dealing with relative, rather than absolute, distances between a population origin and the parks in the system.
In its more sophisticated form, an index of relative attractiveness has been introduced into the gravity formula (Thompson, 1967; Wennergren and Nielsen, 1970). This index enables the prediction of spatial distribution as a function of both distance and parks' characteristics.

The intervening opportunities model (Clark and Roeske, 1969), which is related to the gravity model, specifically attempts to overcome the problem of substitution among recreational areas. It also originates from a law of physics, having been derived from a law determining the distribution of lengths of paths of molecules in a gas. It assumes that the probability of a trip originating in area \( i \) and terminating in site \( j \) will be directly related to the total number of recreational opportunities in area \( j \) and inversely related to the number of opportunities closer to area \( i \) than to area \( j \). In principle, it should be possible to weigh each intervening site by an attraction index in addition to locational weight (distance from area \( i \)). The intervening opportunities model may be considered superior to the simpler gravity model because it fixes relative prices in a spatial context. It considers the relative location of a subset of intervening parks in the system, such as those lying off a given highway connecting areas \( i \) and \( j \).

The micro-structural aspects of the recreational system are neglected by both the gravity and the intervening opportunities models. These aspects stem from the complex relationships in a system of several population centers and recreational sites. They include travel patterns in a network of interconnecting roads as well as a consideration of the externalities in the system.

3. Outdoor recreation system as an electrical analog

The electrical network analog represents another attempt to adapt physical models to recreational planning. It aims to incorporate some of the basic phenomena which the econometric approach or the gravity models either cannot handle, or handle in a manner unsatisfactory from a planner's point of view. These aspects of recreational planning are traffic volumes at various links of the road networks, the presence of intervening and competing recreational opportunities, park congestion, and the dynamic spatial distribution of recreationists over the park system.

The way these aspects would be represented by the elements of the electrical analog is described below.
3.1. Modelling the road network in the analog

The analog attempts to simulate relevant components of the real system. Hence, the entire road network relevant to the recreation system is incorporated. If distance were the single factor determining recreational traffic between population centers and parks then a model of the road network would consist of the set of shortest routes between any pair of population centers and parks. In fact we know that different recreationists choose various routes to reach a given park from their common place of residence, or that the same people may choose different routes on different trips. This variability is accounted for by differences in tastes, in road quality or its scenic characteristics, as well as by the sheer excitement of new and unfamiliar routes. Thus, the entire relevant part of the road network is represented. Its basic building block is the link — a section of a road between two intersections (or nodes). Although the recreationist setting out upon a trip does not necessarily see it as consisting of a series of ‘links’, he is aware — as he must make some decision in that regard — of the various possibilities for substitution among routes with respect to time, distance, congestion, quality, scenery and the like. The breakdown of the network into a system of links and intersections enables us to represent this process in the simulator.

Distance is assumed to represent money expenditures, travel time, and disutility, if any, associated with travel. Consequently, a longer road, ceteris paribus, implies a higher degree of impedance to travel. This impedance-like effect is simulated by a resistor in an electrical circuit. The resistance-like effect of a road could be attributed to its length, width, surface quality, or scenery. A longer road would therefore be modelled by an electrical resistor at a higher resistance level.

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3 In the following discussion we assume that cars constitute the basic units of the recreation system on the demand side. This implies that all recreation flow is described in terms of motorized traffic. No distinction is made between private cars and public transportation (buses), as the latter constitute only a small fraction of the total volume of the recreation traffic.

4 Assume that road link $i$’s resistance, $R_i$, is characterized by its length, $D_i$, by the average speed of cars travelling on it, $S_i$, and by the total travel expenditure (fuel, oil, etc.) associated with it. Specifically, assume that the following relationship holds:

$$R_i = \alpha_1 \left( \frac{D_i}{S_i} \right) + \alpha_2 D_i W_i + \alpha_3$$

where $W_i$ is average expenditure per mile per car; $\alpha_1$, $\alpha_2$ and $\alpha_3$ are positive parameters, and $D_i/S_i$ is the time necessary to travel that link. If we now let $\alpha_3 = 1$, then the resistance—distance ratio, $R_i/D_i$, becomes a constant, given the speed and average expen- (continued on next page)
An elementary formula in electrical circuit theory, Ohm's Law, serves to characterize traffic behaviour over a highway link between two intersections. The equation describes behaviour across a resistor:

\[ V = IR, \]

where \( V \) is potential difference (voltage), \( I \) is current, and \( R \) is resistance. The parameter \( R \) relates to road characteristics; \( V \) and \( I \) are variables related to travel activity.

The electrical current could be likened to a flow of recreationists resulting from their propensity to recreate, associated with a 'pull' due to a park's attractiveness or the 'push' of their wanting to get away from their place of residence and work. The actual number of people engaged in outdoor recreational activities is measured by a potential difference, or voltage, between two points in the system. This includes measurements at the park as well as on the roads leading to it. The meaning of eq. (1) is, therefore, that for any given \( I \), the number of travelling units, \( V \), actually using a link is inversely proportional to its length and quality, both represented by driving time, given by \( R \). If \( I \) is expressed in cars per hour (or any other appropriate travelling unit), and \( V \) in number of cars, the dimension of \( R \) is hours, which measure the length of time required, on the average, per car, to travel that link.

3.2. Modelling population centers

The population centers from which all recreational travel originates, are represented by current sources. This source 'creates' the overall demand for outdoor recreation within the system. This flow of recreationists is determined by the size of the population center, by demographic and socioeconomic characteristics of that population, and by calendar time - seasons, holidays, and weekends. In the present model this volume is exogenously predicted by standard prediction methods, based on records of actual recreational travel flows.

3.3. Modelling the parks

The third component of the modelled system is the recreational

\[ (\text{continued from previous page}) \]

\(^4\) ditire per mile. As long as these two values (speed and expenditure) remain fixed, \( R_i \) is proportional to \( D_i \), as assumed in the text. Conversely, changes in \( S_i \) or \( W_i \) would be represented by changes in \( R_i \), given the length of the \( i \)th road link. The above formula for \( R_i \) assumes linear substitution between time and money expenditures. Other possible substitution relationships are discussed in a paper by Cesario and Knetsch (1970).
site or park. A park is characterized by certain physical qualities, service facilities, and a range of recreational activities allowed by these qualities and facilities. These characteristics determine the park’s attractiveness to potential recreationists. An important upper limit on these characteristics is capacity: each park can accommodate only a finite number of recreationists in any activity within a given time period. Because some activities are interrelated, the total number of recreationists that a park can accommodate in any time period is not the sum total of all the activities’ capacities but a lower figure (e.g., campers usually occupy picnic tables, and so do bathers). The attractiveness and capacity aspects of parks are elaborated below.

3.3.1. Attractiveness

The choice among competing parks depends on the interacting influences of two factors, distance and attractiveness. Parks having similar facilities but not equidistant from a population center, attract people in an inverse relation to their distance. However, dissimilar parks, equidistant from the population center, attract recreationists in direct relation to their relative attractiveness. Distance has already been referred to as a proxy for demand price. However, parks’ characteristics should also enter into the demand schedule, since they influence the quality of the recreational product. This specifically applies to sites of unique natural or scenic merits, as well as to the number and scale of the various facilities provided at any park (Hill and Shechter, 1971).

The attractiveness of a park could therefore be expressed in an analogous manner to price, as it involves a quality factor affecting the demand schedule. Quality is customarily reflected in the quantity-price space by a shift in the demand curve for the product. That is, more units will be bought at the same price due to the higher quality of the product. This approach implies that park attractiveness could be modelled similarly to roads, but inversely. While resistance, $R$, indicated ‘friction’ or impedance to travel, a parameter $G$, such that $G = 1/R$, should serve for park attractiveness. Thus, a smaller resistance value implies higher attractiveness values and vice versa. An increase in a park’s quality is therefore presumed analogous to a decrease in price which, in our model, is reflected by reduced travel costs through their proxy variable — distance. The empirical determination of $G$ is discussed in the section on the application of the model to the Judean Hills region.
Thus, following Ellis (see Chubb, 1967; and Ellis, 1966), a park would be represented by a resistor $R_i$ whose resistance, $R_i$ is a serial addition of two resistors, namely

$$R_i = R_{li} + \frac{1}{G_i}$$

(2)

where $R_{li}$ is associated with the road link between park $i$ and the nearest highway, and $G_i$ represents attractiveness. In addition, $R_{li}$ serves to simulate the effect of entrance fees. If, for example, there are no entrance fees, and the park is contiguous to the highway, $R_{li} = 0$. Entrance fees would then be reflected in positive values for $R_{li}$ and, hence, in $R_i$.

3.3.2. Capacity

A factor which constrains the volume of recreationists of any given park is its capacity. While price (distance) and quality (attractiveness) influence demand, capacity affects the supply schedule in this system.

In the short run, the physical characteristics of a park, particularly those related to land area, the size of water bodies and their shorelines, and the scale of service facilities, determine the park’s capacity in terms of the maximum number of people it can serve.\(^5\) If up to that point the supply is characterized by constant marginal cost, at that point it turns completely vertical. Such a supply curve, however, does not take into account social costs arising from congestion externalities. When the quantity demanded at the given expenditure levels (travel costs, entrance fees, and other intangible costs) approaches or exceeds capacity limitations, crowding ensues. Congestion has opportunity costs either because the affected parties could have stayed home had they known in advance the true benefits forthcoming from the trip or because they could avoid the crowded park by travelling an extra distance to another one. These external effects cause the marginal cost curve to gradually rise even before physical capacity is reached, thereby rationing the available product at a higher real cost.\(^6\) The final equilibrium point (demand and supply intersection) would consequently lie to the left of the equilibrium point which neglects the

\(^5\) Biologists and landscape planners would define an ecological carrying capacity, not necessarily equal to human capacity, for parks having unique natural or landscape qualities.

\(^6\) Capacity is sometimes defined as the point (on the quantity axis) where congestion externalities ensue.
crowding effect. The final equilibrium point is actually the one that physical models should strive to predict.

In order to incorporate into the model capacity and congestion phenomena, it is necessary to analyze demand in a dynamic context. This is done by investigating the process of visitor-accumulation in a park, that is, the effect of past flows on present attendance. To this end we use information on duration of recreational activities in the various parks of the system and, having described the accumulation process, we introduce capacity limitations. The presence of both accumulation and capacity, gives rise to the congestion phenomenon.

3.3.3. The process of accumulation

The interaction among the number of people entering and leaving a park, the activities engaged in by those in the park, and the average duration of those activities\(^7\) determine the rate of accumulation.

The electrical element serving to simulate the accumulation effect for any given park is the capacitor, \(C\). It is an energy storing device whose behaviour is described by the following equations:

\[
Q_\tau = \int_0^\tau I_t \, dt
\]  

\[
V_\tau = \frac{Q_\tau}{C}
\]

where \(Q_\tau\) is charge at time \(\tau\); \(V_\tau\) is potential (voltage) at time \(\tau\); \(I_t\) is current at time \(t\); and \(C\) is the capacitance parameter. Behaviour at each park is simulated by two such equations.

The inflow of recreationists, \(I_t\), is determined, in part, by the park’s attractiveness parameter, \(G\). Since \(I_t\) represents the total number of people entering the park at any period of time and not the rate of net addition (that is, subtracting the outflow at \(t\), \(Q_\tau\) sums the gross inflow up to time \(\tau\). For example, it could represent the total number of visitors to a park during an entire weekend. \(V_\tau\), however, would represent the net accumulation effect, i.e., the number of people actually present in the park at time \(\tau\). This number is proportional to

\(^7\) The average length of time required for a nature walk might be shorter than that needed for a picnic or a swim. Empirical observations bear that out (Chubb, 1967, pp. 102–122).
the cumulative number of visitors, \( Q_r \), the factor of proportionality being \( 1/C \). A park characterized by activities with short (long) duration would be assigned a large (small) value of \( C \). For a given volume of daily visitors and in the presence of capacity limitations this park would be associated with a large (small) turnover of recreationists and by less (more) tendency to congestion conditions.

In appendix A we show the use of \( C \) to represent the accumulation effect, yielding the expression

\[
C = N \frac{1 - \sigma}{1 - \sigma^N}
\]

where \( \sigma \geq (A-1)/A \). \( A \) is the average duration of stay at a given park. \( \sigma \) is the percentage of people who remain at the park one additional period, and \( N \) is the number of time intervals in the whole recreation period.

Thus, by simulating these phenomena – the inflow and outflow of recreationists and the behaviour of those engaged in the various activities in terms of their duration, as implied by the \( C \) values, it is possible to represent the final equilibrium points, i.e. the number of recreationists in the different parks of the system as a function of time. The accumulation effect is described in the numerical example below.

3.3.4. Congestion

We have attempted to describe the rudimentary effects of accumulations using the fact that each park is characterized by a mix of activities with unique average duration. This information is used to formulate the process of visitors’ accumulation in the park, and, in turn, in the presence of capacity limitations, helps explain the congestion effect.

It is possible to depict the physical capacity as a breakdown voltage, \( V_{\text{max}} \), of a capacitor, according to the following relationship:

\[
V = \begin{cases} 
\frac{Q}{C} & \text{for } V < V_{\text{max}}, \\
V_{\text{max}} & \text{for } V \geq V_{\text{max}}.
\end{cases}
\]  

(5)

After the park reaches its physical capacity, \( V_{\text{max}} \), we may assume the following regarding additional visitors: (a) they return home;
(b) they move to another park; and (c) they enter the park and cause congestion.

Since we are concerned with the congestion aspect, we only deal with the last two possibilities. These possibilities can be described by a switch which is activated when \( V_{\text{max}} \) is reached. This, in turn, activates a second resistor, \( R_2 \), of a very high resistance level (fig. 1).

The combined resistance value, \( R_1 + R_2 \), causes either complete rejection of visitors (possibility (b)), or, if \( R_2 \) is not very large, a much reduced flow of visitors (possibility (c)). The former case implies that people move to other parks in the system, while the latter implies that people enter and crowding increases.\(^5\)

3.4. Operating characteristics of the overall system

Eqs. (1) and (4) can be looked upon as basic behavioral building blocks of the outdoor recreation system describing the elementary relationships which characterize the behavior of system components—roads, parks and population origins. In addition to these elementary formulae, it is necessary to state the relationships which hold among the components in generating the overall system behavior.

The electric circuit network which governs the performance of the analog conforms to Kirchoff’s First Law:

\[
\sum_{m \in M} I_m = 0
\]  \hspace{1cm} (6)

\(^5\) In fact, of course, subjective perception of congestion might have begun at an earlier point.
where $I_m$ is the current in link $m$, $M$ is the set of all links associated with node $M$. Eq. (6) states that the sum of all currents in and out of a given node should equal zero. In terms of the recreational system it implies that the number of cars entering any road intersection via the various road links leading into it is equal to the number of units leaving this intersection. The same interpretation applies to any given park: the number of people entering it should equal that which leaves it.

A second relationship governing the behaviour of the analog, is a theorem defining potential difference across a set of links. It states (Berge and Ghoula Houri, 1965, pp. 141–154) that a vector $V = (V_1, V_2, \ldots, V_n)$ is a potential difference if, and only if, there exists a function $v(a)$ defined on the set of nodes such that for any link $k$ between nodes $a$ and $b$ we have

$$V_k = v(a) - v(b).$$

(7)

Finally, we arbitrarily select one of the nodes as a reference node and assign it zero voltage. This procedure ascertains that any potential difference created at the origin by the current source will cause a flow towards the destinations, as is true in the modelled system.

3.4.1. An illustration

Consider the following simple system. It consists of two population centers, two parks, and six highway links. The roads and parks are

![Fig. 2. A schematic representation of a simple recreational system.](image-url)
characterized by resistors only. We assume instant flow and no upper limits on physical capacities. Such a system is assumed in the RECSYS model. The electrical analog consists of two current sources, and eight resistors. Fig. 2 shows the schematic representation of the system.

Using eqs. (1) and (7), we represent each of the six road links and the two parks as follows:

\[
I_j = G_j [v(m_j) - v(n_j)] , \quad j = 1,2 \tag{8}
\]

\[
I_j = \frac{1}{R_j} [v(m_j) - v(n_j)] , \quad j = 3,\ldots,8
\]

where \(m_j\) and \(n_j\) are two nodes associated with the end points of link \(j\), and where links 1 and 2 correspond to the two parks. Since \(G_j = 1/R_j\), eq. (8) can be written as

\[
I_j = G_j [v(m_j) - v(n_j)] , \quad j = 1,2,\ldots,8 . \tag{9}
\]

The node equations satisfying the First Law of Kirchoff, eq. (6), are:

node 1 : \(I_1 - I_6 - I_3 = 0\)

node 2 : \(I_2 - I_4 - I_5 = 0\)

node 3 : \(I_3 + I_4 - I_7 - I_8 = 0\)

node 4 : \(I_6 + I_8 - I_e = 0\)

node 5 : \(I_5 + I_7 - I_g = 0\) \tag{10}

where \(I_e\) and \(I_g\) are the currents emanating from the current sources. Furthermore, using eq. (9) and letting \(v(0) = 0\) (the reference node), we express the links' currents as follows:

\[
I_1 = G_1 v_1 \quad I_5 = G_5 (v_5 - v_2)
\]

\[
I_2 = G_2 v_2 \quad I_6 = G_6 (v_4 - v_1)
\]

\[
I_3 = G_3 (v_3 - v_1) \quad I_7 = G_7 (v_5 - v_3)
\]

\[
I_4 = G_4 (v_3 - v_2) \quad I_8 = G_8 (v_4 - v_3)
\] \tag{11}

where, for convenience, we write \(v_n = v(n)\).